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Effect of temperature on photon–photon entanglement in a nonlinear nanocavity

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HIGHLIGHTS

- Thermal photon–photon entanglement in a nonlinear nanocavity is investigated.
- Photon–photon couplings occur via centrosymmetric dielectrics inside the cavity.
- The system's ground state is shown to be unentangled (separable).
- Photon–photon entanglement exhibits maxima at certain controllable temperature.
- The photonic states are entangled even at extremely high temperatures.

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ABSTRACT

In this paper we study the properties of photon–photon thermal entanglement occurring in a nonlinear optical cavity. The cavity is in thermal equilibrium with a reservoir at a temperature T, so that the bimodal photonic states are determined by the Boltzmann factor. The nonlinear cavity couples the two modes via first and third order susceptibilities. The structure of the total Hamiltonian enables us to develop a computational scheme for determining the energy eigenstates and eigenvalues. Consequently, the thermal density matrix, the negative eigenvalues of the partially transposed one and, thereby, the negativity, as a measure of photon–photon free entanglement, are computed. Our results show that the negativity vanishes at absolute zero, indicating that the ground state is separable. As the temperature increases, the negativity exhibits a maximum, at a certain temperature, and then asymptotically vanishes. Moreover, we demonstrate how the maximal entanglement as well as the temperature at which it occurs, is characterized by the properties of the medium. The roles of nonlinearities on such characteristics are also discussed in detail.

1. Introduction

It is by now well established that quantum entangled states are of crucial importance in the field of quantum information theory, particularly, quantum teleportation, quantum computation, etc. [1–5]. It is thus fundamental to explore means of generation and characterization of entanglement suitable for such applications. In this regard, "cavity quantum electrodynamics" has been proposed as a vivid candidate for the realization of entangled states, with potential applications in quantum information processing [6,7]. To be more specific, this approach has been employed to form atom–atom [8,9], atom–photon [10,11], or photon–photon [12,13] entanglements. In this approach, the photon–photon entanglement occurs

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through an intermediary atom embedded inside the cavity [14]. There is, however, an unavoidable fact that the cavity is normally filled by dielectrics whose nonlinear properties indeed produce photon–photon entanglement [12]. Parenthetically, we may add that such nonlinearities have been used for applications in optical switching [15], frequency selection and modulation [16], nonlinear quantum couplers [17], nonlinear quantum scissors [18,19], etc. [20–23]. In all of the aforementioned applications the central point is the fact that, even at very low intensities (a few photons), nonlinear interactions are indeed agitated [24]. The behavior of photonic states, thereby the photonic entanglements, in the presence of such dielectrics is thus mainly governed by Kerr effects (self and cross couplings) and a linear interaction [12,18–23,25,26]. It is therefore the main purpose of the present report to investigate the effect of these interactions on the thermal photon–photon entanglements.

To maintain the entanglements, it is essential to avoid de-coherence due to the leakage of cavity modes and the interaction with the environment, acting as a heat reservoir [27,28]. As a result and in addition to the nonlinear effects, the temperature is also expected to drastically influence the formation and characterization of photon–photon entanglement. It is thus another goal of the present work to investigate the effect of temperature, in addition to nonlinearities, on the entanglement between photons inside a cavity filled by a nonlinear dielectric. At this point we call attention to the fact that in our treatment temperature influences the sub-systems on the same footing, so that we encounter "free" (distillable) [29–31] entanglement in what follows. The dielectric is taken to be centrosymmetric, characterized by Kerr self-actions, Kerr cross-action and a linear coupling [12]. Moreover, it is assumed that the photons are in thermal states with probabilities specified by the Boltzmann factor. In order to determine the Boltzmann factor, we show that the total Hamiltonian of the system is block-diagonal whose dimensions depend on the photonic excitations. The structure of the Hamiltonian then enables us to compute numerically the energy eigenvalues which, in turn, leads to the corresponding photonic thermal states. The results are then used to determine the degree of thermal (mixed states) photonic free entanglement via the negativity.

Quantification of entanglement for bipartite compositions in pure states is by now settled through the concepts of von Neumann entropy, concurrence, etc. [32,33]. On the other hand, quantification of entanglement for mixed states (the case in hand) is not as simple and has encountered subtleties [34,35]. However, for 2 × 2 or 2 × 3 bipartite quantum systems the measure of negativity, amongst others [36–38], accurately quantifies the degree of entanglement [35]. Briefly, if the density matrix of a bipartite quantum system can be written as $\rho = \sum_i p_i\rho_i^A \otimes \rho_i^B$, where $p_i \geq 0$, $\sum_i p_i = 1$ and $\rho_i^{A,B}$ represents the density matrices for the subsystems, then the system is separable (disentangled), otherwise it is entangled [39]. This criterion is then expressed in terms of the negativity, as a measuring device, in the following manner [35,39]. The elements of ρ is given by $\langle a, b|\rho|\acute{a}, \acute{b}\rangle$ where $\{|a\rangle\}$ and $\{|b\rangle\}$ form the orthonormal basis for each subsystem. It has been shown that for the composite system to be separable, it is necessary that the partially transposed density matrix, defined as, $(\rho^{PT})_{a,b;\acute{a},\acute{b}} = \langle \acute{a}, b|\rho|a, \acute{b}\rangle = \langle a, \acute{b}|\rho|\acute{a}, b\rangle$, has no negative eigenvalues [39]. Conversely, if ρ^{PT} possessed even a single negative eigenvalue, then the quantum system would be entangled (inseparable). Quantitatively, this criterion may be expressed in terms of the negativity, defined as, $N = \sum_n \mathrm{Max}(0, -\lambda_n)$, where λ_n's are the eigenvalues of ρ^{PT}. It then follows that the state of the composite system is separable (disentangled) when the negativity is null, otherwise it is entangled [35]. In spite of the fact that the concept of negativity forms a necessary and sufficient condition for 2 × 2 or 2 × 3 bipartite systems [40], it is generally trusted for higher dimensional quantum entities [41,42]. It is worth pointing to the fact that the aforementioned discussion holds true for free, versus bound, entanglement [29–31], which is indeed applicable to the case under consideration in the present report. We thus calculate the negativity as a measure of (free) entanglement between the thermally induced photons inside the cavity. From the results thus obtained the role of controlling agents, namely, the Kerr parameters and the linear coupling, as well as the temperature, is determined and discussed.

This paper is organized as follows. In Section 2 the bimodal nonlinear field–field Hamiltonian, along with a Casimir operator that commutes with the total Hamiltonian, are presented. We proceed to examine the matrix representation of the Hamiltonian, with due attention to the Casimir operator, and show that it is block-diagonal. After giving the numerical approach to evaluate the negativity as a function of temperature in Section 3, we will present our numerical results by several figures in Section 4. Interpretation of the result and physical reasons are also discussed in this section. We highlight the more important aspects of the article in the concluding section.

2. The physical model

Considering a two-mode quantized electromagnetic field inside a cavity filled with a nonlinear medium, the Hamiltonian of the system, neglecting multiphoton exchanges, can be written as [12],

$$H = \hbar\sum_{i=1}^{2}\omega_i\left(a_i^\dagger a_i + \frac{1}{2}\right) + \hbar\sum_{i=1}^{2}\chi_i(a_i^\dagger)^2 a_i^2 + \hbar\overline{\chi}a_1^\dagger a_1 a_2^\dagger a_2 + \hbar\lambda(a_1^\dagger a_2 + a_2^\dagger a_1), \tag{1}$$

where the first term describes the Hamiltonian of the free bimodal photons with frequencies ω_i, the second and third terms are due to Kerr couplings and the last term describes the linear coupling of the two modes. As was described in the introduction, χ_i and $\overline{\chi}$ arise from the third order susceptibility (self and cross actions, respectively) while λ arises from the first order susceptibility. It is observed from Eq. (1) that the first three terms determine the bare energy eigenvalues whose spacing prevents the entanglement, while the last term, the linear coupling, assists the entanglement. Moreover, it is easy to show that the (Casimir) operator, $N = \sum_{i=1}^{2} a_i^\dagger a_i$, whose eigenvalues, N_e, giving the total number of photonic excitations,