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An analysis of the radial flow in the heterogeneous porous media based on fractal and constructal tree networks

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ABSTRACT

In this paper, an analysis of the radial flow in the heterogeneous porous media based on fractal and constructal tree networks is presented. A dual-domain model is applied to simulate the heterogeneous porous media embedded with a constructal tree network based on the fractal distribution of pore space and tortuosity nature of flow paths. The analytical expressions for seepage velocity, pressure drop, local and global permeability of the network and binary system are derived, and the transport properties for the optimal branching structure are discussed. Notable is that the global permeability (K_n) of the network and the volume fraction (f_n) occupied by the network exhibit linear scaling law with the fractal dimension (D_p) of channel diameter by $\log K_n \sim 0.46 D_p$ and $\log f_n \sim 1.03 D_p$, respectively. Our analytical results are in good agreement with the available numerical results for steady-state soil vapor extraction and indicate that the fractal dimension for pore space has significant effect on the permeable properties of the media. The proposed dual-domain model may capture the characteristics of heterogeneous porous media and help understanding the transport mechanisms of the radial flow in the media.

1. Introduction

The radial flow toward a well is of pivotal importance for the oil/water reservoir engineering and has been a long-steady research subject [1–3]. The reservoir properties often show heterogeneities due to differences of grain size, shape, sorting, packing, and orientation or diagenesis [4]. Flow to well in randomly heterogeneous formations is of central interest to hydrogeologists and petroleum engineers. In the past, the transient pressure tests and uniform or nonuniform flows were often investigated. There are, however, very few micro-geometrical models involved in the heterogeneity of the media, and few theoretical analyses are concerned with the transport properties of such flows.

The formation and development of hydrofractures is one of the main mechanisms for permeability in fluid-filled heterogeneous reservoirs. It has also been shown that, due to the fluvial processes and material deposition, the highly heterogeneous porous media often contain connected paths of high-permeability behaving as preferred pathways for fluid flow and chemical migration, and these pathways have the dominant effect on the flow transport [5–8]. The preferential flow paths (including fractures) may connect each other and form a branching channel network and may significantly influence on macroscopic properties for many field settings [8,9]. This is particularly true in the case of radial flow in the underground reservoir [10]. Therefore, the identification of the hydraulic characteristics and transport properties of the heterogeneous porous media requires the development of a specific model that accounts for the medium heterogeneity and the geometrical structure of the connected channel network.

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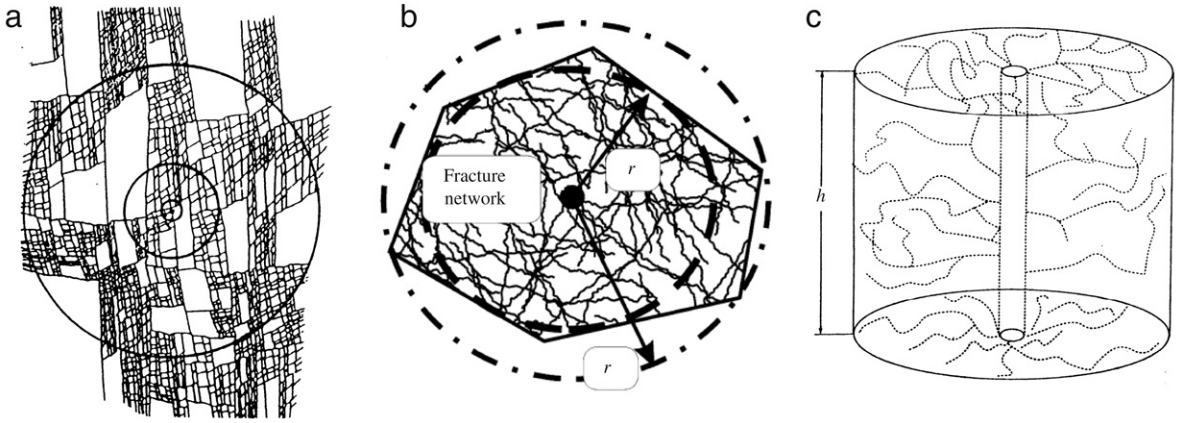

Fig. 1. (a) Fracture networks [29], (b) sketch of the fracture network around a well [30], and (c) a schematic of flow in porous media [31].

Bejan et al. [11,12] proposed the constructal theory and constructal tree network with optimization of flow and heat transfer. Constructal designs come from the balance between two mechanisms, channels with stream flow and diffusion (seepage) across the interstices. Recently, Lorente and Bejan [13] indicated that the features of heterogeneous porous media with multiple scales and nonuniform distribution are similar to tree-like flow network, and the tree-like flow network can also be obtained by using the constructal theory in the heterogeneous porous media. One may also find the similar structures in nature such as trees, leaves, river basins, mammalian circulatory and respiratory systems, world-wide web, Internet and social networks etc. It has been shown that the structures of these tree-like networks may have the fractal tree/branching structures which can be space filling [14], ensuring minimal dissipation [11–13,15–20] and pumping power [11–13,18–21], and showing significantly more error (variability) tolerant than other structures, and therefore the structures have an evolutionary advantage [22].

The former investigations were all limited to transport properties of one single tree-like network *themselves* [11–27]. However, real flow paths in heterogeneous porous media are actually composed of huge amount of different branching networks. The diameter distribution of the original channels (0th level in branching networks) may show to be fractal distribution. Furthermore, a dual-domain/dual-porosity model (also called the binary system in this work) may be more suitable for characterizing real heterogeneous porous media. The concept about dual-domain/dual-porosity was initially proposed by Barenblatt et al. in 1960 [28]. This concept assumes the simultaneous existence of two continuous and porous systems, which have distinctly different porosities and permeabilities. Only the more permeable system (generally the fissure/fracture system) can produce to the well. While the less permeable (generally the porous matrix), which has a much higher storage of fluid, feeds fluid into the fissure/fracture system.

Fig. 1(a) and (b) respectively show a typical fissure/fracture system [29] and the fractures around a well which can be approximated as circular shape [30], which forms a network consisting of connected channels with higher permeability serving as preferential pathways for fluid flow, and the surrounding matrix is a low-permeability material with low permeability/porosity. Fig. 1(c) is a schematic of flow toward a well [31]. Thus, this type of media forms a dual-domain/dual-porosity medium. Both of the fracture network [29–32] and the matrix porous media [31,33] can be represented conveniently and with a good approximation by fractal. Therefore, we may build a dual-domain/dual-porosity medium, with the constructal tree networks embedded in an isotropic fractal porous medium. The tree network describes the connected capillary/channel network around a well in a heterogeneous reservoir, see Fig. 2. The tree network can be constructed by constructal theory [11–13] from one point to a circle to characterize the connected channel network and investigate the transport properties of radial flow toward a well in heterogeneous porous media. And also, the tortuosity of the flow pathways is taken into account in the tree network model. We will derive the local and global distributions of the seepage velocity, pressure drop, and effective permeability of the dual-domain system. As argued by Ronayne and Gorelick [8], this channel-matrix conceptualization is a useful heuristic dual-domain model for studying dynamical processes in many natural systems that contain branching conduit geometries. It should be pointed out that the tree network from one point to a circle of preferential flow paths presented in this study should be viewed as an idealized representation and only intended to represent one of the several possible/plausible configurations of aquifer architectures in highly heterogeneous environments [7].

This paper is organized as follows. In Section 2, we depict our dual-domain model and construction of the embedded tree channel network from one point to a circle characterizing the preferential flow paths. In Section 3, we study the transport of radial flow toward a well and calculate the seepage velocity and effective permeability of the dual-domain heterogeneous porous system. The important results and discussions of the proposed model are given in Section 4. The concluding remarks on the transport properties of the radial flow in the dual-domain system toward a well and a few comments regarding the possible applications of the present analysis are presented in Section 5.