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Resemblances and differences in mechanisms of noise-induced resonance

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Abstract

Systems showing stochastic resonance (SR) or coherent resonance (CR) share some features, in particular, the nearly periodic character of the signal. We show that in spite of this resemblance the different underlying dynamics can be detected in experimental data by studying the histogram of the inter-spikes times and some statistical properties like two-time correlation functions. We discuss its possible relevance for climate modeling.

Keywords: Stochastic resonance; Coherent resonance; Climate modeling

1. Introduction

The mechanism of stochastic resonance (SR) was initially introduced as a possible explanation for climate changes on long time scales [1]. During the last two decades

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it has been applied to a wide class of systems such as analog circuits, neurobiology, ring lasers, systems with colored noise, etc.; for a review see Ref. [2].

The prototypical system showing SR, which is also the original one used to model climate changes, is the stochastic differential equation

$$\frac{dx}{dt} = -\frac{\partial V(x,t)}{\partial x} + \sqrt{2D}\eta , \tag{1}$$

where η is a Gaussian white noise with $\langle \eta(t) \rangle = 0$ and $\langle \eta(t)\eta(t') \rangle = \delta(t - t')$, D measures the noise intensity and $V(x,t)$ is a double-well potential with a time-periodic term

$$V(x,t) = \frac{x^4}{4} - \frac{x^2}{2} + Ax\cos(2\pi t/T) . \tag{2}$$

In the case of a stationary potential, i.e., $A = 0$, the jumps between the two minima at $x = -1$ and $x = 1$ are independent events whose probability distribution is approximately Poissonian [3]. Using simple arguments based on the Kramers exit-time formula [4], it can be shown that there is a range of values of D, T and A where SR is present, i.e., the jumps between the two minima (close to -1 and $+1$ if A is sufficiently small) are strongly synchronized with the forcing and that the probability distribution function (PDF) of the jumping time τ has a relatively sharp peak around T [1,2].

The phenomenon of SR provides one example of the nontrivial role that noise can play in dynamical systems with an external periodic forcing. Besides SR, there exist other examples of the "constructive role" of noise, e.g., one can have a synchronization of trajectories generated by different initial conditions and the same noise realization [5]. Our interest will focus on cases where noise can enhance periodic behavior, e.g., the so-called coherent resonance (CR) and the noise-induced dynamics in systems with time delay (ND).

The phenomenon of CR [6] has been found in models describing excitable systems that occur in different fields like chemical reactions, neuronal and other biological processes [7,8]. The prototypical stochastic differential equation used in this case is the FitzHugh–Nagumo system defined by

$$\varepsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y , \tag{3}$$

$$\frac{dy}{dt} = x + a + \sqrt{2D}\eta , \tag{4}$$

with $\varepsilon \ll 1$ so that the time evolution of x is much faster than that of y. For $|a| > 1$ there is a stable fixed point; for $|a| < 1$ there is an unstable fixed point and a limit cycle. The cycle consists of two portions of slow motion connected by a fast jump. If $|a|$ is slightly larger than 1 the system is excitable [6], i.e., small deviations from the fixed point may generate large pulses (also called spikes) [9]. Moreover, in this case, one finds that there is a range of values of the noise intensity D such that CR appears, i.e., roughly periodic noise-excited oscillations are present, resembling the SR oscillations [6].