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On the application of nonextensive statistical mechanics to the black-body radiation

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Abstract

Some results for the black-body radiation obtained in the context of the nonextensive statistical mechanics (normalized approach) are analyzed. Since the obtained elsewhere thermodynamic potential can be expressed in terms of Wright's special function a useful asymptotic expansion can be obtained. This expansion allows to consider in a simple way the thermodynamic properties of the black-body radiation away from the Boltzmann–Gibbs limit $q \to 1$. The proposed approximation scheme is physically reasonable for the analysis of the cosmic background radiation. It is shown that while the internal energy remains extensive the application of the concepts of the nonextensive lead to the T^{d+1} Stefan–Boltzmann behavior.

Keywords: Tsallis statistics; Black body radiation; Cosmic background radiation

1. Introduction

In the last few years many papers [1–11] have been published on the application of nonextensive statistical mechanics (NSM) [12] to the black-body radiation. May be this is not the simplest application of the general theory but one believes that here eventual discrepancies between theory and experiment are more easily observable

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than elsewhere. Our belief is that rigorous and exact results have an instructive role in this field. The exact expression for the corresponding partition function has been obtained in Refs. [5,9,10]. The studies presented in Refs. [1,2,5–8,11] employ the unnormalized approach [13], while those in Refs. [9,10] used the normalized one [14,15].

Irrespective of the approach used one can see that the final expressions one has at hand are too cumbersome and obscure the underlying physics. In this situation an approximation that tries to make the exact expressions more simple and transparent is preferable. However, the real benefit from the exact treatment without a well-defined range of validity of the used approximation seems to be doubtful.

In the context of NSM, two approximating schemes have been used in the black-body studies namely asymptotic approximation [1] and factorization approximation [3,4]. Let us note that the more appropriate approximations are limited to simply computing $(1-q)$ corrections (see Refs. [7] and [10] and references therein) since the Boltzmann–Gibbs limit $q \to 1$ leads to great simplifications. The validity range and usefulness of both approximations have been discussed in Ref. [16]. The presented discussion demonstrated that $(1-q)$ expansion and the factorization approximation are not useful for a system with N (the number of harmonic oscillators) very large and arbitrary q. More precisely these approximations seem reasonable only if $(1-q) \ll 1$ [16]. In this situation for a system with infinitely many degrees of freedom (as black-body radiation is) an asymptotic expansion with range of validity away from the usual Boltzmann–Gibbs limit $q \to 1$ are of significant interest.

A well-estimated approximation in this field also would be useful, since recently this issue has been a matter of a debate in the literature [17–19].

The aim of the present study is to illustrate another possibility for the simplification of basic expressions not related to the small value of $(1-q)$ or to the factorization scheme. It is based on the fact that in both unnormalized and normalized approaches the intricate sums that appear in the theory may be presented [20] in terms of the Wright function with well-studied analytical properties [21]. The normalized approach is believed to be superior to the unnormalized one. That is why the former will be used in this study.

First we shall introduce some basic notions. The radiation field in a large cavity can be considered to consist of a denumerably infinite set of electromagnetic oscillators corresponding to the various quantum states $\mathbf{k}$ in a d-dimensional box. The oscillator frequencies, $\omega_i = ck_i$, are related to the total energy E by $E = \Sigma_i n_{i,\varepsilon} \hbar\omega_i$, where $n_{i,\varepsilon}$ is the number of oscillator quanta with frequency ω_i and polarization ε, c is the light speed, $\hbar$ is the Planck constant and $k_i = |\mathbf{k}_i|$. The Boltzmann–Gibbs partition function Z_1, for a large volume V, can be written as

$$Z_1 = \exp(A_d)\,, \tag{1}$$

where

$$A_d = \frac{\Gamma(d)\zeta(d+1)2\tau_d}{(4\pi)^{d/2}\Gamma(d/2)}\left(\frac{k_B T}{\hbar c}\right)^d V\,. \tag{2}$$