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Dynamics of a stochastic Holling II one-predator two-prey system with jumps

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HIGHLIGHTS

- We consider a stochastic Holling II one-predator two-prey model with jumps.
- The sufficient conditions for the extinction and persistence of the solution are established.
- We analyze how the jump noise affects the population dynamics.

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ABSTRACT

In this paper, a stochastic Holling II one-predator two-prey system with jumps is investigated. Firstly, the properties of the solution, such as the existence and uniqueness of the global positive solution, stochastic ultimate boundedness and the pathwise estimation are studied. Then we mainly establish the sufficient conditions for the extinction and persistence in the mean of the solution. Results show that positive jump noise is advantageous to the system while negative jump noise is disadvantageous. Finally, a numerical example is introduced to illustrate the results.

1. Introduction

In ecology, living in groups is a widespread phenomenon for population species and has attracted a great attention in many different areas [1–5]. Ref. [1] has discussed the mechanisms that govern the evolution and maintenance of grouping behavior. By applying the self-organization theory to the group-living vertebrates, authors in Ref. [2] attempted to understand the dynamics of the collective behaviors. In Ref. [3], authors pointed out reality mining (the processing and analysis of machine-sensed data on the social behavior of animals or humans) can make it possible to investigate the social behavior of almost human populations in extraordinary detail. In Ref. [4], the authors reviewed the observations and the basic laws describing the essential aspects of collective motion. In Ref. [5], the author adopted a collective behavioral model immersed in a fluid to predict the two-dimensional space–time evolution of the predator–preys system, and elucidated how the hydrodynamics impacts the predator's attack.

However, aforementioned papers mainly discussed the social behaviors of the population species. In ecology and mathematical ecology, it is also important to study the interrelationship between species and their environment by using mathematical models. There are three main types of interaction: predator–prey, competition and mutualism. And there have been

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growing interests on the dynamical behavior of the population species living in groups, such as the extinction, permanence, stability and so on. In particular, the predator–prey interaction have been studied extensively. In all predator–prey interaction, the functional response, which refer to the number of prey eaten per predator per unit time as a function of prey density, is a key element [6]. In Ref. [7], Holling gave three different kinds of functional response to model the predation, that is, Holling type I, II, and III functional response. A classical Holling II predator–prey system with two completing preys and one predator can be expressed as follows:

$$\begin{aligned}
\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} &= x_1(t)\left(r_1 - a_{11}x_1(t) - a_{12}x_2(t) - \frac{a_{13}y(t)}{1+x_1(t)}\right),\\
\frac{\mathrm{d}x_2(t)}{\mathrm{d}t} &= x_2(t)\left(r_2 - a_{21}x_1(t) - a_{22}x_2(t) - \frac{a_{23}y(t)}{1+x_2(t)}\right),\\
\frac{\mathrm{d}y(t)}{\mathrm{d}t} &= y(t)\left(-r_3 + \frac{a_{31}x_1(t)}{1+x_1(t)} + \frac{a_{32}x_2(t)}{1+x_2(t)} - a_{33}y(t)\right),
\end{aligned} \tag{1.1}$$

where $x_i(t)$ $(i = 1, 2)$ are the population sizes of prey species and $y(t)$ is the population size of predator species at time t, $r_i > 0$ $(i = 1, 2, 3)$ are the intrinsic growth rates or death rate, $a_{ii} > 0$ $(i = 1, 2, 3)$ stand for the intraspecies interaction, $a_{ij} \geq 0$ $(i \neq j)$ represent the effect of species j upon the growth rate of species i. In recent years, qualitative analysis of the predator–prey model (1.1) with Holling II functional response and its extension have been studied by many authors, for example, see Refs. [8–16].

However, in the real world, population systems are always affected by various environmental noises. The dynamics of stochastic population systems has attracted much attention since it is of practical importance. As a matter of fact, many good results on the dynamical behaviors of stochastic models driven by Brownian motion have been reported, for example, see Refs. [17–27]. But sometimes population systems may suffer some sudden environmental perturbations, such as toxic pollutants, earthquakes, hurricanes, floods and so on [28]. These sudden environmental shocks will cause jumps on population dynamics and make the solution discontinuous, so stochastic population systems with jumps become a research hotspot recently. Many good research papers about stochastic models with jumps have been reported, see Refs. [28–33].

Motivated by above facts, in this paper, we consider the following stochastic two-prey one-predator system with Holling II functional response:

$$\begin{aligned}
\mathrm{d}x_1(t) &= x_1(t)\left(r_1 - a_{11}x_1(t) - a_{12}x_2(t) - \frac{a_{13}y(t)}{1+x_1(t)}\right)\mathrm{d}t + \sigma_1 x_1(t)\mathrm{d}B_1(t) + \int_{\mathbb{Y}} c_1(u)x_1(t^-)N(\mathrm{d}t, \mathrm{d}u),\\
\mathrm{d}x_2(t) &= x_2(t)\left(r_2 - a_{21}x_1(t) - a_{22}x_2(t) - \frac{a_{23}y(t)}{1+x_2(t)}\right)\mathrm{d}t + \sigma_2 x_2(t)\mathrm{d}B_2(t) + \int_{\mathbb{Y}} c_2(u)x_2(t^-)N(\mathrm{d}t, \mathrm{d}u),\\
\mathrm{d}y(t) &= y(t)\left(-r_3 + \frac{a_{31}x_1(t)}{1+x_1(t)} + \frac{a_{32}x_2(t)}{1+x_2(t)} - a_{33}y(t)\right)\mathrm{d}t + \sigma_3 y(t)\mathrm{d}B_3(t) + \int_{\mathbb{Y}} c_3(u)y(t^-)N(\mathrm{d}t, \mathrm{d}u),
\end{aligned} \tag{1.2}$$

where $x(t^-)$ is the left limit of $x(t)$, $B_i(t)$ is mutually independent Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}\}_{t\geq 0}, \mathbb{P})$, σ_i^2 is a positive constant representing the intensity of the white noise, $i = 1, 2, 3$. N is a Poisson random measure. Let λ be the characteristic measure of Poisson random measure N, and $\lambda(\mathbb{Y}) < \infty$, where $\mathbb{Y}$ is a measurable subset of $(0, \infty)$. Define the compensated random measure by $\widetilde{N}(\mathrm{d}t, \mathrm{d}u) := N(\mathrm{d}t, \mathrm{d}u) - \lambda(\mathrm{d}u)\mathrm{d}t$.

Compared with the literature, contributions and novelties of the current work are as follows:

(1) We introduce jump noise to the one-predator two-prey model with Holling II functional response.
(2) We establish the sufficient conditions for the extinction and persistence of the solution to system (1.2).
(3) We analyze how the jump noise affects the population dynamics.

The remaining part of this paper is organized as follows. In Section 2, we give some fundamental properties of solution to system (1.2). Section 3 deals with the survival analysis of this model. In Section 4, we introduce some simulation figures to illustrate our theoretical results.

2. Properties of the solution

Throughout this paper, $\mathbb{R}_+ := (0, \infty)$. For convenience and simplicity, let

$$b_i = r_i - \frac{\sigma_i^2}{2} + \int_{\mathbb{Y}} \ln(1 + c_i(u))\lambda(\mathrm{d}u), \quad i = 1, 2,$$

$$b_3 = -r_3 - \frac{\sigma_3^2}{2} + \int_{\mathbb{Y}} \ln(1 + c_3(u))\lambda(\mathrm{d}u).$$

$$\langle f(t)\rangle = t^{-1}\int_0^t f(s)\mathrm{d}s, \qquad \langle f(t)\rangle^* = \limsup_{t\to\infty} t^{-1}\int_0^t f(s)\mathrm{d}s, \qquad \langle f(t)\rangle_* = \liminf_{t\to\infty} t^{-1}\int_0^t f(s)\mathrm{d}s.$$

For the rest of this paper, we also require that