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Non extensive thermodynamics for hadronic matter with finite chemical potentials

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HIGHLIGHTS

- Partition function for nonextensive quantum gas at finite chemical potential.
- Transition line between confined and deconfined regimes.
- Relevant thermodynamical function in nonextensive statistics.
- Possible application to neutron-star studies.

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ABSTRACT

The non extensive thermodynamics of an ideal gas composed by bosons and/or fermions is derived from its partition function for systems with finite chemical potentials. It is shown that the thermodynamical quantities derived in the present work are in agreement with those obtained in previous works when $\mu \leq m$. However some inconsistencies of previous references are corrected when $\mu > m$. A discontinuity in the first derivatives of the partition function and its effects are discussed in detail. We show that at similar conditions, the non extensive statistics provide a harder equation of state than that provided by the Boltzmann–Gibbs statistics.

1. Introduction

One of the main concerns in the study of ultra-relativistic collisions is the investigation of the quark–gluon plasma (QGP) properties. In this regard the thermodynamical aspects of the plasma is specially interesting due not only to the possibility of studying the deconfinement process but also because it can give important information to other fields, like hydrodynamical models of the QGP, cosmological models of the early Universe and models of massive objects in astrophysics. The non extensive statistics has been applied to a large number of problems since the seminal paper by C. Tsallis in 1988 [1]. An updated list of applications and studies on the subject can be found in Ref. [2]. In High Energy Physics (HEP) nonextensivity was introduced by Bediaga, Curado and Miranda (BCM) in 2000 [3]. In that work the authors used the well-known transverse momentum (p_T) distribution from Hagedorn's theory [4] and formally substituted the exponential function by the q-exponential function that appears in the Tsallis statistics. With the new distribution obtained by BCM it is possible to describe the whole p_T-distribution measured in HEP experiments.

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A large number of works reporting the use of the non extensive formalism have been published since the BCM work, all of them showing a good agreement with experiments (see for instance Refs. [5–9]). More recently a non extensive generalization of Hagedorn's theory [10–16] was developed in Ref. [17] showing that not only a limiting (or critical) temperature, T_o, exists but also that there is an entropic index, q_o, which characterizes the hadronic systems at least for the confined regime. Also a new formula for the hadronic mass spectrum in terms of T_o and q_o was derived from that theory and in Ref. [18] it is shown that this formula can describe quite well the spectrum of known hadronic states with masses from the pion mass up to ∼2.5 GeV. The non extensive self-consistent theory [17] imposes much more restrictive tests to the applicability of the Tsallis statistics in HEP and a number of analysis of experimental data [18–21] have shown that the theoretical predictions are in agreement with the experimental findings.

The non extensive thermodynamics of hadronic matter was already explored for null chemical potential (μ) systems in Ref. [22] and compared with Lattice-QCD data, showing a reasonable agreement. In the present work we extend the thermodynamics to finite chemical potential systems, which is of importance in the study of nucleus–nucleus collisions and of astrophysical objects. An important class of compact objects are protoneutron stars. The understanding of their evolution in time from the moment they are born as remnants of supernova explosions until they completely cool down to stable neutron stars, has been a matter of intense investigation. All sorts of phenomenological equations of state (EOS), relativistic and non-relativistic ones, have been used to describe protoneutron star matter. These EOS are normally parameter dependent and are adjusted so as to reproduce nuclear matter bulk properties, as the binding energy at the correct saturation density and incompressibility as well as ground state properties of some nuclei [23–25]. The present work provides the necessary formalism for the investigation of how the non extensive statistics affects stellar matter.

The paper is organized as follows: in Section 2 we introduce the partition function and show that it is in agreement with the p_T-distribution used in previous works [3,17,21,26]; in Section 3 we derive the thermodynamical functions of interest and compare with previous results in the literature; in Section 4 we establish the phase transition line between confined and deconfined regimes in the $T \times \mu$ diagram and discuss the properties of the EOS of hadronic matter; and finally we present our conclusions in Section 5.

2. Partition function for non extensive thermodynamics

We next outline the main formulas necessary for the development and application of the non-extensive formalism to hadronic matter. Consider the exponential function defined as

$$\begin{cases} e_q^{(+)}(x) = [1+(q-1)x]^{1/(q-1)}, \quad x \geq 0, \\ e_q^{(-)}(x) = \dfrac{1}{e_q^{(+)}(|x|)} = [1+(1-q)x]^{1/(1-q)}, \quad x < 0. \end{cases} \tag{1}$$

We define the q-logarithm as

$$\begin{cases} \log_q^{(+)}(x) = \dfrac{x^{q-1}-1}{q-1}, \\ \log_q^{(-)}(x) = \dfrac{x^{1-q}-1}{1-q}, \end{cases} \tag{2}$$

which would correspond to the inverse function of the q-exponential if $\log_q(x)$ were defined by $\log_q^{(+)}(x)$ for $x \geq 1$ ($\log_q^{(-)}(x)$ for $x < 1$). In the following we make use of these two definitions of the q-exponential, but do not consider in general their definition regimes in this way. It follows straightforwardly that

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}x} e_q^{(+)}(x) = [e_q^{(+)}(x)]^{2-q}, \\ \dfrac{\mathrm{d}}{\mathrm{d}x} e_q^{(-)}(x) = [e_q^{(-)}(x)]^{q}, \\ \dfrac{\mathrm{d}}{\mathrm{d}x} \log_q^{(+)}(x) = x^{q-2}, \\ \dfrac{\mathrm{d}}{\mathrm{d}x} \log_q^{(-)}(x) = x^{-q}. \end{cases} \tag{3}$$

The relations above are used many times in the following and specially for the derivation of the identities:

$$\frac{\mathrm{d}}{\mathrm{d}x} \log_q^{(-)}\left(\frac{e_q^{(+)}(x) - \xi}{e_q^{(+)}(x)} \right) = \xi \left[\frac{1}{e_q^{(+)}(x) - \xi} \right]^q, \tag{4}$$

for $x \geq 0$, and

$$\frac{\mathrm{d}}{\mathrm{d}x} \log_q^{(+)}\left(\frac{e_q^{(-)}(x) - \xi}{e_q^{(-)}(x)} \right) = \xi \left[\frac{1}{e_q^{(-)}(x) - \xi} \right]^{2-q}, \tag{5}$$