Physica A 421 (2015) 249–260

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Jamming transitions and the effect of interruption probability in a lattice traffic flow model with passing

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HIGHLIGHTS

- A lattice model is proposed by considering the interruption effect with passing.
- To describe the effect of interruption probability, linear analysis is performed.
- Jamming transitions are explored by reduction perturbation technique.
- The effect of reactive coefficients on phase diagram is investigated.
- The effects of interruption probability on traffic flow are also examined.

ARTICLE INFO

Article history:
Received 7 August 2014
Received in revised form 11 November 2014
Available online 22 November 2014

Keywords:
Traffic flow
Interruption
mKdV equation
Chaotic flow

ABSTRACT

A new lattice hydrodynamic model is proposed by considering the interruption probability effect on traffic flow with passing and analyzed both theoretically and numerically. From linear and non-linear stability analysis, the effect of interruption probability on the phase diagram is investigated and the condition of existence for kink–antikink soliton solution of mKdV equation is derived. The stable region is enhanced with interruption probability and the jamming transition occurs from uniform flow to kink flow through chaotic flow for higher and intermediate values of non-interruption effect of passing. It is also observed that there exists conventional jamming transition between uniform flow and kink flow for lower values of non-interruption effect of passing. Numerical simulations are carried out and found in accordance with the theoretical findings which confirm that the effect of interruption probability plays an important role in stabilizing traffic flow when passing is allowed.

1. Introduction

Due to rapid modernization in the last few decades, the increase in vehicular traffic on roads causes serious problems in terms of traffic congestion, traffic safety, traffic efficiency and energy consumption. Therefore, the analyses of various complex traffic phenomena have become an attentive topic of deliberation and research for scientists and engineers, nowadays. Many different approaches [1–23] are introduced to investigate the properties of traffic congestion and understand the nonlinear phenomena of traffic flow, such as non-equilibrium phase transitions and nonlinear waves. Nagatani [24], firstly, introduced a lattice hydrodynamic model in which drivers adjust their velocity according to the observed headway. Afterwards, many extensions have been carried out by considering different factors like backward effect [25], lateral effect of the lane width [26], passing effect [27], anticipation effect of potential lane changing [28] and optimal current difference effect [29].

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http://dx.doi.org/10.1016/j.physa.2014.11.037

In real traffic, drivers often observed various traffic interruptions e.g., accident, signal light, tolling station, pedestrian on the road and according to that accelerate or decelerate their vehicles to prevent from collision. Generally, various traffic interruptions are related to traffic conditions and road structure. Some traffic interruptions like accidents always occur with some probabilities and produced complex phenomena on roads. The interruption on the roads results into increasing traffic flow on existing roadways inevitably results in a rise in congestion which leads to delays, decreasing flow, higher fuel consumption and has negative environmental effects. Therefore, interruption on the roads affect the traffic flow in huge amount and the negative impact of interruptions can be significantly reduced by the proper use of incident management. So, there is a need to study mathematical models for analyzing the effect of interruption on traffic flow from transportation point of view. However, above mentioned traffic models are not able to evaluate the effect of various traffic interruption probabilities as traffic interruption factors are not taken into account. In this direction, Tang et al. [30] proposed a macro model by considering the effect of interruption probability based on the relationship between micro and macro variables. Also, Tang et al. [31] introduced a car following model by considering the effect of interruption probability. Recently, Peng [32] proposed a new lattice hydrodynamic traffic flow model by considering the effect of traffic interruption probability. But the important aspect of passing has not been considered in Peng's model. Also, to avoid traffic interruptions on road, drivers try to overtake or change the lane to prevent from congestion so that drivers can move with their optimal speed. In addition, many traffic flow models [33–35] are proposed by considering the driver's contribution in different aspects.

Motivated by the recent advancements in hydrodynamic traffic models, this paper deals with the development and analysis of a new lattice hydrodynamic model with the consideration of the traffic interruption probability when passing is allowed. The rest of the paper is organized as follows: in Section 2, a new lattice model is proposed by introducing the effect of interruption probability when passing is allowed. In Sections 3 and 4, linear and nonlinear stability analysis are carried out to investigate the impact of interruption probability when passing is allowed. Finally, to validate the theoretical findings, numerical simulations are performed out in Section 5 and conclusions are given in Section 6.

2. A modified Lattice hydrodynamic model

To analyze the density wave of traffic flow on unidirectional roads, the first lattice hydrodynamic model which incorporates the ideas of car following models and macroscopic models is proposed by Nagatani [24] and given as

$$\rho_j(t+\tau) - \rho_j(t) + \tau\rho_0\left[\rho_j(t)v_j(t) - \rho_{j-1}(t)v_{j-1}(t)\right] = 0, \tag{1}$$

$$\rho_j(t+\tau)v_j(t+\tau) = \rho_0 V(\rho_{j+1}(t)), \tag{2}$$

here j indicates site-j on a one-dimensional lattice; ρ_j and v_j, respectively, represent the local density and velocity at site-j at time t; ρ_0 is the average density; a $(=1/\tau)$ is the sensitivity of drivers; $V(.)$ is the optimal velocity function.

In real traffic, driver always tries to overtake his leading vehicles when he finds congestion in front of him to maintain their optimal speed. To incorporate this idea, Nagatani [27] proposed a lattice model by assuming that when the traffic current on site-j is larger than on site-$j+1$, a passing occur and is proportional to the difference between the optimal currents at site-j and $j+1$. The continuity equation remains preserved while the evolution equation is modified by looking at the difference of traffic current on site-j and $j+1$. The modified evolution equation taking passing into account is

$$\rho_j(t+\tau)v_j(t+\tau) = \rho_0 V(\rho_{j+1}(t)) + \gamma[\rho_0 V(\rho_{j+1}(t)) - \rho_0 V(\rho_{j+2}(t))], \tag{3}$$

here γ is the passing constant. In the above model, the jamming transition occurs among uniform traffic flow, through chaotic flow, to the kink flow when passing is allowed [27]. For $\gamma = 0$, the model reduces to Nagatani's model [24] in which the jamming transition occurs between uniform traffic flow to the kink flow.

On a highway, drivers encounter various interruptions due to accident, traffic signal and tolling station, and these interruptions lead to complexity in the traffic flow dynamics. The above models cannot study the complex phenomena resulting from some traffic interruption factors. Using lattice hydrodynamic approach, Peng [32] incorporate the effect of interruption probability by considering the interruption at site-$j+1$ with probability p_{j+1}. The flux at site-$j+1$ will be very slow due to interruption and can be taken as approximately zero. The proposed evolution equation with traffic interruption probability is given as

$$\rho_j(t+\tau)v_j(t+\tau) = \rho_0 V(\rho_{j+1}) + \gamma_1 p_{j+1}(-Q_j) + \gamma_2(1-p_{j+1})(Q_{j+1} - Q_j), \tag{4}$$

where $Q_j = \rho_j v_j$, represents the traffic flux at site j at time t; p_{j+1} is the interruption probability at site-$j+1$ due to some traffic interruption factors. In the above model, it is found that stable region enhances with increasing the traffic interruption probability. However, the above models did not consider the effect of passing. In view of this, we proposed a new evolution equation with consideration of interruption probability on a single-lane highway when passing is allowed and given as follows:

$$\rho_j(t+\tau)v_j(t+\tau) = \rho_0 V(\rho_{j+1}) + \gamma_1 p_{j+2}[\rho_0 V(\rho_{j+1}(t))] + \gamma_2(1-p_{j+2})[\rho_0 V(\rho_{j+1}(t)) - \rho_0 V(\rho_{j+2}(t))], \tag{5}$$

where p_{j+2} is the probability that traffic flow of lattice site-$j+2$ is interrupted, γ_1, and γ_2 are the reactive coefficients of interruption and non interruption terms and the interruption term indicates that driver adjusts the vehicle after sensing the