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A Tsallis entropy-based redundancy measure for water distribution networks

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HIGHLIGHTS

- Tsallis entropy is employed for deriving a redundancy measure for water distribution networks.
- The Tsallis entropy-based redundancy measure is found to have a one-to-one relation with the Shannon entropy-based measure.
- Tsallis entropy-based measure can be employed for design of water distribution systems.
- The Tsallis entropy-based redundancy values show that the redundancy increases with increasing number of loops in the network.

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ABSTRACT

A measure of redundancy inherent in the layouts of water distribution networks is developed using the Tsallis entropy. Both the local redundancy at a node and the global redundancy due to the redundancies at adjacent nodes are derived. The redundancy measure is applied to layouts reported in the literature and is compared with the Shannon entropy-based measure. Using the values reported in the literature, it is shown that there is almost a one-to-one relation between the Shannon entropy and the Tsallis entropy-based redundancy and reliability and this relation allows to specify the reliability of a network whose redundancy is known. Hence, the redundancy measure can also be employed in the water distribution network design.

1. Introduction

Design of a water distribution system is inherently a multi-objective optimization problem, for it entails competing objectives, including the minimization of head losses, cost, risk, and departures from specified values of water quantity, pressure and quality; and the maximization of reliability [1]. However, it is not uncommon to formulate the design problem as a single objective optimization problem where the system capital and operational costs are minimized and at the same time the laws of hydraulics are satisfied and the targets of water quantity and pressure at demand nodes are met. Fundamental to either type of optimization is reliability [2–4]. Many studies deal with the reliability of overall water supply systems [5–9], while others deal with the reliability of within water distribution networks [10–14]. This study is concerned with the reliability of water distribution networks. The need for reliability stems from uncertainties in consumer demand, fire flow requirements and their locations, pumping systems failure, inefficient storage, pipe failures and their locations, valve leakages and their locations, and reduced capacity due to sedimentation. Goulter [15] argued that the shape or layout of a network determined the level of reliability that can be imposed on the network.

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Approaches to the optimization of water distribution networks have been either entirely deterministic or stochastic. Examples of deterministic optimization include studies of Goulter and Coals [10], Sue et al. [11], Lansey et al. [16], Goulter and Bouchart [14] among others that focused on reliability within an optimization framework with respect to the hydraulic performance of the network under a range of mechanical failures and demands. Likewise, Ekinci and Konak [17] presented an optimization strategy for water distribution networks, based on the minimization of head losses for least cost design, whereas Eiger et al. [18] presented a 2-stage decomposition model for optimization of water distribution networks. Todini [19] developed a technique for looped water distribution network design using a resilience index.

In the stochastic approach to optimization for design of water distribution networks, Giustolisi et al. [20] presented a multi-objective optimization scheme incorporating the uncertainty of nodal water demands and pipe roughness, with the objective to minimize costs and maximize hydraulic reliability. Kwon and Lee [21] analyzed reliability of pipe networks using probability concepts focusing on transient flow which can cause failure of the water distribution system. On the other hand, entropy theory has been applied to develop measures for water distribution network reliability. In a review of explorative uses of entropy, Templeman [22] discussed application of entropy to water supply network analysis. Using the Shannon entropy, Awumah [23] and Awumah et al. [5,6] developed redundancy measures for water distribution systems. In a discussion of the study by Awumah et al. [6], Xu and Jowitt [24] remarked their entropy-based measure needed further investigation. Fundamentally, redundancy in a water distribution network means that demand points or nodes have alternative supply paths for water in the event that some links go out of service. In a redundant network there is sufficient residual capacity to meet water flow requirements. Thus, redundancy is a characteristic of a water distribution system and is related to its reliability. In order to ensure reliability the water distribution network design must incorporate some amount of redundancy.

Tanyimboh and Templeman [25] described methods using entropy for computing the most likely flows in the links of the networks with incomplete data. Tanyimboh and Templeman [26] presented an algorithm for calculating the maximum entropy flows for single source networks. Chen and Templeman [27] presented entropy-based methods for mathematical planning. Perelman and Ostfeld [28] developed a cross-entropy-based algorithm for optimal design of water distribution systems. Subsequently, Perelman et al. [1] extended the cross-entropy based algorithm to multi-objective optimization for water distribution systems design. The extended method coupled the cross-entropy algorithm [29] and some features of multi-objective evolutionary techniques [30].

Survey of literature shows that in water distribution networks design, either the Shannon entropy [31] or the cross-entropy [32] has been employed. However, the Tsallis entropy [33] does not seem to have been used in the evaluation of reliability or design of water distribution systems. It may therefore be interesting to explore the use of the Tsallis entropy in the development of redundancy measures for water distribution networks, for it possesses a number of interesting properties and encompasses the Shannon entropy as a special case. The objective of this study was, therefore, to develop a methodology, using the Tsallis entropy, for evaluating the redundancy and in turn reliability of a water distribution system. The paper is organized as follows. Introducing the problem of water distribution network redundancy and its estimation in this section, the Tsallis entropy is briefly outlined in Section 2 and redundancy measures based on this entropy are developed in Section 3. A relationship between redundancy and reliability is developed in Section 3.7, followed by the evaluation of the Tsallis entropy-based measures are developed in Section 4. The paper is concluded in Section 5, followed by an Appendix on the Tsallis entropy for an ensemble of systems.

2. Tsallis entropy

Tsallis [33] introduced a new type of entropy which is now referred to as the Tsallis entropy, S, which can be expressed for a discrete random variable X with probability distribution $P = \{p_i,\ i = 1, 2, \ldots, N\}$ where p_i are probabilities for $X = x_i,\ i = 1, 2, \ldots, N$, as

$$S(x) = \frac{K}{m-1}\left(1 - \sum_{i=1}^{N} p_i^m\right) = \frac{K}{m-1}\sum_{i=1}^{N} p_i(1 - p_i^{m-1}) \quad (1)$$

where m is a real number, K is a conventional positive constant (needed to keep the units of S consistent) taken as unity without loss of generality, and N is the number of values X takes on. S describes the uncertainty associated with p_i and in turn with X. If $(1-p_i^{m-1})/(m-1)$ is considered as a measure of uncertainty, then Eq. (1) represents the average uncertainty of X. The Tsallis entropy is a non-extensive entropy and reduces to the Shannon entropy if exponent m in Eq. (1) tends to unity. It can also be said that for $m \to 1$, Eq. (1) reduces to the Boltzmann–Gibbs statistics. S is maximum for all values of m in the case of equiprobability. S is maximum if $m > 0$ and is minimum if $m < 0$. Like the Shannon entropy, the Tsallis entropy satisfies the additivity property for independent systems. Because of these and other properties, the Tsallis entropy has been applied to describe a wide range of areas, including turbulence, fractility and non-extensivity, scaling, anomalous diffusion, and complexity. Tsallis [34] provided an extensive review of historical background and the present status. In a subsequent study, Tsallis [35] presented a comprehensive review of the construction and physical interpretation of nonextensive statistical mechanics, with particular reference to the Tsallis entropy. These applications have been primarily in physics, although many of them relate to hydrological processes and will therefore have relevance in hydrological analysis and modeling.