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Lattice hydrodynamic modeling of two-lane traffic flow with timid and aggressive driving behavior

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HIGHLIGHTS

- A lattice model is proposed by considering driver's characteristics.
- The effect of driver's characteristics is examined by linear stability analysis.
- Jamming transitions are analyzed through mKdV equation.
- The effect of the proportion of aggressive and timid drivers is investigated.
- The effect of anticipation coefficient on driver's characteristics is studied.

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ABSTRACT

In this paper, a new two-lane lattice hydrodynamic traffic flow model is proposed by considering the aggressive or timid characteristics of driver's behavior. The effect of driver's characteristic on the stability of traffic flow is examined through linear stability analysis. It is shown that for both the cases of lane changing or without lane changing the stability region significantly enlarges (reduces) as the proportion of aggressive (timid) drivers increases. To describe the propagation behavior of a density wave near the critical point, nonlinear analysis is conducted and mKdV equation representing kink–antikink soliton is derived. The effect of anticipation parameter with more aggressive (timid) drivers is also investigated and found that it has a positive (negative) effect on the stability of two-lane traffic flow dynamics. Simulation results are found consistent with the theoretical findings which confirm that the driver's characteristics play a significant role in a two-lane traffic system.

1. Introduction

Due to the rapid increase of automobiles on road, the traffic congestion became more and more serious problem and attracted much attention of scientists and researchers because of its complex mechanism, recently. It not only increases energy consumption and emissions but it also imposes safety hazards. Therefore, in past few decades various traffic models, such as car-following models, cellular automaton models, lattice hydrodynamic models, gas-kinetic models, and fluid-dynamic models have been developed to understand the dynamics of traffic flow and investigate the properties of traffic jams [1–38]. Among them, the lattice hydrodynamic model firstly proposed by Nagatani [20], which incorporates the idea of the microscopic optimal velocity model, is convenient for analyzing the density wave in traffic flow. Modified Korteweg–de

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Vries (mKdV) equation is derived to describe the traffic congestion in terms of kink density wave near the critical point. Subsequently, this modeling approach was widely referred and extended to study various nonlinear phenomenon present in real traffic flow like backward effect [21], lateral effect of the lane width [22], anticipation effect of individual driving behavior [23] and explicit driver's physical delay [24] etc.

Most of the aforementioned models focus mainly on some traffic phenomena only on a single-lane. These models cannot be applied to fully describe the characteristics of traffic on road networks which consist of two or more lanes, since they do not consider the lane changing behavior. For this reason, Nagatani [28] further extended his model to a two-lane traffic system and investigated the impact of lane changing on the stability of traffic flow. Later, to improve the performance of two-lane Nagatani's model more realistically, some modifications have been made by incorporating different factors like optimal current difference [29], and flow difference effect [30] etc. Peng [31] analyzed the effect of driver anticipation in two-lane traffic system. Gupta and Redhu [32] developed a new model to investigate the effect of drivers anticipation in sensing relative flux (DAESRF) for two-lane system. Very recently, Zhang et al. [33] extended two-lane traffic model to incorporate the effect of driver's delay.

In real traffic, the driver always adjusts his/her velocity according to observed traffic situation in the surroundings and estimates his/her driving behavior. To capture this complex phenomenon, few efforts [27–38] have been made in the past. Unfortunately, none of these models can be used to explain the influence of driver's characteristics (timid or aggressive) on traffic flow. Most of highways comprise of multi-lanes, so it will be more adequate to investigate this effect on a two-lane system with the consideration of potential lane changing. This motivates us to develop a two-lane lattice model by incorporating the effect of driver's characteristics.

In this paper, a more realistic lattice model with the consideration of driver's behavior on two-lane traffic system is presented. The rest of the paper is organized as follows: In Section 3, the stability condition of traffic flow is derived by means of linear stability theory. To describe the propagation behavior of traffic jams, Section 4 is devoted to the nonlinear analysis in which mKdV equation is computed near the critical point. Numerical simulations are carried out to validate the theoretical findings in Section 5 and finally, conclusions are drawn in Section 6.

2. A new model

To describe the traffic phenomena on an unidirectional single-lane highway, Nagatani [20] in 1998, firstly, proposed the simplest lattice hydrodynamic model given as

$$\partial_t \rho_j + \rho_0(\rho_j v_j - \rho_{j-1} v_{j-1}) = 0, \tag{1}$$

$$\partial_t(\rho_j v_j) = a[\rho_0 V(\rho_{j+1}) - \rho_j v_j] \tag{2}$$

where ρ_0 is the average density; ρ_j and v_j, respectively, represent the local density and velocity at site j at time t. $V(.)$ is the optimal velocity function which is a monotonically decreasing function having an upper bound and an inflection point at critical density. The idea is that the variation in traffic flow ρv at site j is determined by the difference between the actual flow at site j and the optimal flow $\rho_0 V(\rho_{j+1})$ at the next site $j+1$.

While describing the traffic flow on road networks, multi-lane models are found to be more appropriate as most of the road networks are made up of more than one-lane. In this regard, Nagatani [20] further extended single lane lattice model to describe traffic phenomenon on two-lane by incorporating lane change effect in the lattice version of continuity equation. The lane changing on a two-lane highway occurs only in the following two possibilities.

Case (1): If $\rho_{2,j-1}(t) > \rho_{1,j}(t)$, i.e. the density at site $j-1$ on the second lane is higher than that at site j on the first lane the lane changing occurs from the second lane to the first lane and the lane changing rate will be proportional to their density difference as follows: $\gamma|\rho_0^2 V'(\rho_0)|\left(\rho_{2,j-1}(t) - \rho_{1,j}(t)\right)$.

Case (2): If $\rho_{1,j}(t) > \rho_{2,j+1}(t)$, i.e. the density at site j on the first lane is higher than that at site $j+1$ on the second lane, the lane changing occurs from the first lane to the second lane and the lane changing rate will be proportional to their density difference as follows: $\gamma|\rho_0^2 V'(\rho_0)|\left(\rho_{1,j}(t) - \rho_{2,j+1}(t)\right)$.

Here, $\rho_{1,j}(t)$ and $\rho_{2,j}(t)$ are the densities on the first and the second lane, respectively and γ is a fixed dimensionless coefficient. The proportionality constant $\left(\gamma|\rho_0^2 V'(\rho_0)|\right)$ is chosen in such a way that it becomes dimensionless. Based on the above lane changing rules, the continuity equation for two-lane traffic can be obtained in the same fashion as in Ref. [28] and is given by

$$\partial_t \rho_j + \rho_0(\rho_j v_j - \rho_{j-1} v_{j-1}) = \gamma|\rho_0^2 V'(\rho_0)|(\rho_{j+1} - 2\rho_j + \rho_{j-1}), \tag{3}$$

where $\rho_j = \frac{\rho_{1,j}+\rho_{2,j}}{2}$ represents the local density at site j and $\rho_j v_j = \frac{\rho_{1,j}v_{1,j}+\rho_{2,j}v_{2,j}}{2}$ represents the local flow at site j for the two-lane system.

In addition, the evolution equation of traffic current on each lane remains unaffected by lane changing. Hence, the evolution equation for two-lane traffic system [28] was incorporated as

$$\partial_t(\rho_j v_j) = a[\rho_0 V(\rho_{j+1}) - \rho_j v_j]. \tag{4}$$

In general, the drivers behavior comprises three equilibrium states being aggressive, normal or timid. In real traffic, different driver's characteristics like aggressive ones, driving fast, and timid ones, driving slowly influences traffic flow to