Physica A 421 (2015) 161–170

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Discrete scale-invariance in cross-correlations between time series

Qin Xiao[a], Xue Pan[a], Mutua Stephen[a,b], Yue Yang[a], Xinli Li[a], Huijie Yang[a,*]

[a] Business School, University of Shanghai for Science and Technology, Shanghai 200093, China

[b] Computer Science Department, Masinde Muliro University of Science and Technology, P.O. Box 190-50100, Kakamega, Kenya

HIGHLIGHTS

- A new method is proposed to detect discrete long-range correlations.
- Sectors in a same stock turn out to have continuous long-range correlations.
- Sectors in different stock markets prove to have discrete scaling invariance.

ARTICLE INFO

Article history:
Received 26 June 2014
Received in revised form 15 October 2014
Available online 18 November 2014

Keywords:
Discrete-scale long-range correlation
De-trended cross-correlation analysis
Stock markets

ABSTRACT

The de-trended cross-correlation analysis (DCCA) is converted to a new form, which turns out to be a periodic function modulated power-law, to evaluate discrete-scale long-range cross-correlation between time series. If the modulator is dominated with one frequency, the derived form will degenerate to a log-periodic power-law. We investigate a total of five important stock markets distributing in different continents. Calculations show that the cross-correlations between different stock markets may hint at log-periodic oscillations. This finding may be helpful for us to evaluate financial state in a global way.

1. Introduction

Economic globalization leads to close correlations between different nations. Effects of important events such as natural and/or social disasters may spread quickly to the whole world. Stock market depends on a large amount of networked factors in a social system, and it is consequently a barometer of social state. Hence, cross-correlations between different stock markets distributing in the world can give us a measure of state of the global sociality.

Previous works show that there exist long-range correlations in stock indices [1–3], namely, auto-correlation function obeys a power-law instead of an exponential decay. Cross-correlations between different stocks have been investigated in detail, but the attentions focus on the relations of stocks in the same stock market [4–6].

Very recently, a de-trended covariance based method, called cross-correlations analysis (DCCA) [7,8], is introduced to evaluate long-range cross-correlations between non-stationary time series. This seminal work stimulates extensive works that form a family of DCCA-based methods, such as the multi-fractal de-trending moving average cross-correlation analysis (MF-X-DMA) [9,10], the multi-fractal de-trended cross-correlation analysis (MF-X-DFA) [11], the multi-fractal height cross-correlation analysis (MF-HXA) [12], and the multi-fractal cross-correlation analysis based on statistical moments (MFSMXA) [13].

* Corresponding author.
E-mail address: hjyang@ustc.edu.cn (H. Yang).

http://dx.doi.org/10.1016/j.physa.2014.11.032

Podobnik et al. [14] apply the method and a test proposed in Ref. [15] to quantify the interaction between the New York Stock Exchange Composite index (NYA) and the Shanghai Stock Exchange Composite index (SSEC), and the NYA and the German financial index DAX. For the absolute values of a daily return series, over the 11-year period from January 4, 2000 to August 15, 2011, it is found that the NYA and the SSEC follow each other only very weakly, while the DAX strongly follows the NYA.

In the present paper, the original version of DCCA is used to measure cross-correlations in stock indices for five influent stock markets distributed in different nations, namely, China, USA, Japan, German, and England. Our contribution is twofold. First, DCCA stands only for continuous scale-invariance. We generalize theoretically it to a new form for discrete scale-invariance, which turns out to be a periodic function modulated power-law. When the modulator is dominated with one frequency, the form degenerates to a log-periodic power-law. Second, calculations may hint at log-periodic oscillations of the cross-correlations between different stock markets. These findings can be used to monitor financial state in a global way.

2. Methods and materials

2.1. De-trended cross-correlations analysis

Let us consider two stationary time series (normalized), $y_i, y_i{}'$, where $i = 1, 2, \ldots, N$. Assuming there exist long-range correlations in and between the two time series, we have several power-law relations for $|n| \gg 1$,

$$\begin{aligned} A(n) &\equiv \langle y_i \cdot y_{i+n} \rangle = C_A \cdot n^{-\gamma}, \\ A'(n) &\equiv \left\langle y'_i \cdot y'_{i+n} \right\rangle = C_{A'} \cdot n^{-\gamma'}, \\ X(n) &\equiv \left\langle y_i \cdot y'_{i+n} \right\rangle = \left(\frac{|n| + n}{2 * |n|} \cdot C_x^{>} + \frac{|n| - n}{2 * |n|} \cdot C_x^{<} \right) \cdot |n|^{-\gamma_x}, \end{aligned} \tag{1}$$

where $\langle \cdot \rangle$ denotes statistical average, $0 \le \gamma, \gamma', \gamma_x \le 1$, and $C_A, C_{A'}, C_A^{>}, C_A^{<}$ constants.

Profiles of the two time series read, $R_s \equiv \sum_{j=1}^{s} y_s$ and $R'_s \equiv \sum_{j=1}^{s} y'_s$, where $s \le N$. A simple computation leads to covariance of the two profiles,

$$\left\langle (R_i - \langle R_i \rangle) \times \left(R'_{i+n} - \left\langle R'_i \right\rangle \right) \right\rangle = nX(0) + \sum_{j=1}^{n-1} (n - j) \cdot [X(j) + X(-j)]. \tag{2}$$

Inserting the following two approximations,

$$\begin{aligned} \sum_{j=1}^{n-1} X(j) &\approx \sum_{j=1}^{n} j^{-\gamma_x} \approx \int_1^n \mathrm{d}x \cdot x^{-\gamma_x} \propto n^{1-\gamma_x}, \\ \sum_{j=1}^{n-1} j \cdot X(j) &\approx \sum_{j=1}^{n} j^{1-\gamma_x} \approx \int_1^n \mathrm{d}x \cdot x^{1-\gamma_x} \propto n^{2-\gamma_x}, \end{aligned} \tag{3}$$

and similar results for sums with $X(-j)$ into Eq. (2), we have an asymptotical relation for $|n| \gg 1$,

$$\left\langle (R_i - \langle R_i \rangle) \times \left(R'_{i+n} - \left\langle R'_i \right\rangle \right) \right\rangle \propto n^{2(1-0.5\gamma_x)}. \tag{4}$$

Generally, time series in real world are non-stationary. Podobnik et al. [7] propose a modification of the above covariance to detect long-range cross-correlations between non-stationary series, called de-trended cross-correlation analysis (DCCA). Let a window with size n slide along R, which results in a total of $N - n$ overlapping segments. Fitting the ith segment with a polynomial function up to a certain order, the regression curve is used here to represent trend for this segment, $\{\tilde{R_s}\},\ s = i, i + 1, \ldots, i + n - 1$. By using the same procedure, one can obtain the trend for the profile R', which read $\{\tilde{R'_s}\},\ s = i, i + 1, \ldots, i + n - 1$. Replacing R and R' in Eq. (4) with their residuals, we have covariance,

$$\begin{aligned} F_{DCCA}^2 &\equiv \frac{1}{N - n} \sum_{i=1}^{N-n} \sum_{s=i}^{i+n-1} \frac{1}{n - 1} (R_s - \tilde{R_s})(R'_s - \tilde{R'_s}) \\ &\propto n^{2(1-0.5\gamma_x)}. \end{aligned} \tag{5}$$

For self-similar processes, the DCCA degenerates to the de-trended fluctuation analysis (DFA) [16], namely, $F_{DFA}^2 \equiv \frac{1}{N-n} \sum_{i=1}^{N-n} \sum_{s=i}^{i+n-1} \frac{1}{n-1} (R_s - \tilde{R_s})^2 \sim n^{2(1-0.5\gamma)} \equiv n^{2H}$, where H is the Hurst exponent.