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# Evidence of economic regularities and disparities of Italian regions from aggregated tax income size data

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## HIGHLIGHTS

- The economical size distribution of the Italian cities over the quinquennium 2007–2011 is discussed.
- The presence of regularities among the data is detected.
- Yearly data are proven to be well fitted by a 2-parameters Lavalette law.
- The occurrence of distortions generated by the outliers (the so-called *king* and *king plus vice-roy effects*) is also explored.
- The analysis is performed either at a national as well as at local (regional and provincial) level.

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## ABSTRACT

This paper discusses the size distribution – in economic terms – of the Italian municipalities over the period 2007–2011. Yearly data are rather well fitted by a modified Lavalette law, while Zipf–Mandelbrot–Pareto law seems to fail in this doing. The analysis is performed either at a national as well as at a local (regional and provincial) level. Deviations are discussed as originating in so called king and vice-roy effects. Results confirm that Italy is shared among very different regional realities. The case of Lazio is puzzling.



## 1. Introduction

The analysis of the ranking of elements belonging to a specific set under a predefined criterion leads to the identification of a best fit[3] curve, through the rank–size theory [4–8] and its applications.

This paper deals with the rank–size rule for the entire set of municipalities in Italy (IT, hereafter) for each year of the quinquennium 2007–2011. The *size* is here given by the *contribution* (so called Aggregated Tax Income, thereby denoted hereafter as ATI) that each city has given to the Italian GDP (data are expressed in Euros); cities are yearly ranked according

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3 All fits, in this communication, are based on the Levenberg–Marquardt algorithm [1–3]; the error bar was pre-imposed to be at most 1%.

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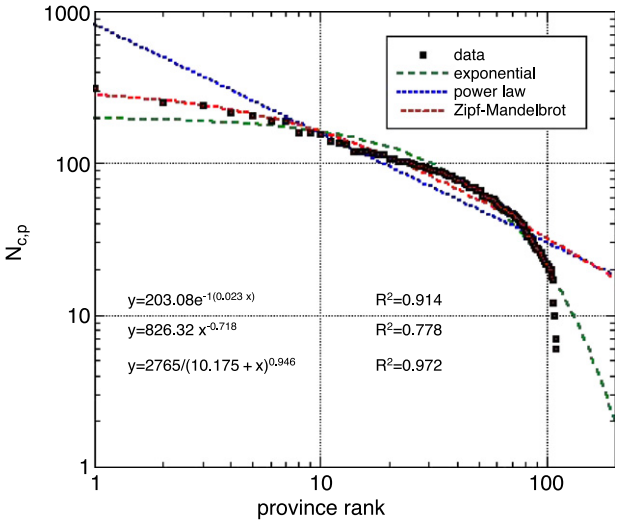


**Fig. 1.** Log–log plot of the number $N_{c,p}$ of cities (8092) per provinces (110), ranked by decreasing order of "importance"; showing fits by a power law, an exponential and a Zipf–Mandlebrot function with the corresponding correlation coefficient. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

to the value of their related ATI. Data are official, and have been provided directly from the Research Center of the Italian Minister of Economic Affairs.

For our investigation, several different directions are followed:

1. the possible law describing the relationship between ranking and ATI is explored. In particular, we show that Zipf, Zipf–Mandlebrot[4] and power laws fail in this doing. A more convincing answer is provided by the Lavalette function [9],
$$y(rank) = K\left(\frac{N \cdot rank}{N - rank + 1}\right)^{-\chi} \equiv \kappa\left(\frac{rank}{N - rank + 1}\right)^{-\chi} \tag{1.1}$$
which has been introduced in 1996 by the biophysicist Daniel Lavalette. Such an analysis is performed not only at the country, but also at the regional and at the provincial level;
2. the distribution of the ATI at the regional level is lengthily explored. In doing so, several cities are shown to exhibit a prominent role in determining a relevant percentage of the national GDP (the so-called *king* and *king plus vice-roy effects*, see Section 4.2 for the details).

In particular, point 1. supports that sometimes data city sizes do not have pure Zipf-type (i.e. a pure power law) links with the corresponding ranks. However, evidence is here shown that some particular subsets of cities may be well described by a statistically appealing Zipf–Mandelbrot law (this is the paradigmatic case of Lazio, an IT region) – a set of considerations postponed for Appendix (Appendix A) in order to let a relatively ordered line of thought guiding the reader in the following sections – without being distracted by the main aims. For the contextualization of these results in the literature, see Section 2.

Also point 2. is in great agreement with an improvement of the best-fit results when some specific subsets of data are considered. In this case, king and king plus vice-roy effects can be appreciated by observing, on displayed plots, that removing the first and sometimes the first set of ranked cities, respectively, leads (not always, but remarkably often) to a more statistically convincing Lavalette curve.

It is important to point out that, to the best of our knowledge, this is the first contribution dealing with the application of the Lavalette curve to the field of urban economics; it was invented and usually applied for bibliometrics studies.

The paper is organized as follows: Section 2 briefly reviews the literature inspiring and connected to the present research. Section 3 contains the description of the data. Section 4 is devoted to the investigation of the whole IT, with the assessment of some rank–size rule fits on yearly basis. This section contains also the ATI ranking analysis at a regional level, with all the plots of the 2-parameter Lavalette functions and the detection of the outliers. Section 5 collects and discusses the findings. Section 6 concludes and offer suggestions for further research lines. Appendix A describes the Lazio case, while Figures and Tables pertaining to the regional data analysis are collected in Appendix B.

## 2. Review of the literature

In the context of New Economic Geography (NEG) – introduced by Krugman [10] and surveyed in Refs. [11–15] – spatial patterns based on geographical agglomerations and dispersions of economic quantities play a fundamental role.

[4] It is sometimes called the Zipf–Mandlebrot–Pareto (ZMP) function.