Physica A 421 (2015) 98–108

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

# Strategy for community control of complex networks

Piao Xiufeng [a], Lv Tianyang [a,b,*], Zhang Xizhe [c], Ma Hui [a]

[a] College of Computer Science and Technology, Harbin Engineering University, Harbin 150001, China
[b] IT Center, National Audit Office, Beijing 100830, China
[c] College of Information Science and Engineering, Northeastern University, Shenyang 110819, China

## HIGHLIGHTS

- It is sufficient and more feasible to control just a part of nodes in many cases.
- Controlling communities is influenced by the remainder of the network.
- The proposed immune nodes facilitate the control of target communities.
- The absolute cost of controlling communities is less when the degree is low.
- The ratio of immune nodes and driver nodes to all nodes increases.

## ARTICLE INFO

*Article history:*
Received 7 June 2014
Received in revised form 19 September 2014
Available online 11 November 2014

*Keywords:*
Complex network
Community control
Partial control
Immune node

## ABSTRACT

The latest researches have explored the topic of complex network control based on the linear system control theory. These researches aim to control all nodes of a network. However, in many cases it is sufficient to control just a part of nodes. Because a community usually plays an independent role in a real complex system, the paper discusses a strategy to control target communities of a complex network. First, the paper shows that the process of controlling target communities will be influenced by their connections with the remainder of the network. Therefore, it is necessary to block the control signals transmitting through these interconnections. Second, the paper proposes a new kind of control nodes that selectively block signal transmissions, termed immune nodes. The immune nodes work along with the driver nodes to facilitate the control of target communities, even if the entire topology of a network is absent. Third, we propose a method to reduce the total number of driver nodes and control nodes by deliberately arranging the matching sequence of nodes. The experiments are carried out on a series of model networks and 11 real networks. The experimental results show that the absolute control cost of controlling communities is less than that of controlling the entire network, when the average degree of the network is low, but the ratio of immune nodes and driver nodes to all nodes increases.



## 1. Background

The goal of complex network control is to drive a complex network from any initial state to a desired state within finite time by inputting just a few control signals.

In 2011, Liu etc. proposed a framework for analyzing complex network controllability based on the control theory of linear systems [1]. Then, researchers of many other fields began studying problems related to complex network control along with

* Corresponding author at: IT Center, National Audit Office, Beijing 100830, China.
*E-mail address:* raynor1979@163.com (T. Lv).

http://dx.doi.org/10.1016/j.physa.2014.10.081
0378-4371/

the cyberneticians [2], such as reducing the number of driver nodes through network structure perturbations [3], controlling edges instead of nodes [4], exploring how the topological features of a network influence its controllability [5], calculating the control energy bounds [6] and carrying out studies of application studies in a specific area [7].

However, these studies aim to control all nodes or all edges of a network, but in many cases it is sufficient to control just a part of a network. For instance, it is sufficient to make the entire Internet unstable by attacking a few key nodes [8]; in the studies of brain neutral network, it is preferred to control some parts of brain to cure diseases [9]. Moreover, it is also difficult to control the entire network, especially when the global topology of a network is absent. In this case, controlling a part of a network would be more feasible and less costly than controlling the whole network.

A community is a relatively independent part of a complex network [10], that is, for a subgraph to be a community the connections amongst its nodes should be denser than those connecting the subgraph with the remainder of the network. And a community usually plays an independent role in a real complex system. For example, a community of WWW may contain the homepages with a similar theme; a community is likely to represent a functional unit in a metabolic or a neural network. Therefore, it is possible to influence a whole system by controlling some of its communities.

This paper treats a community as a basic unit for partial control of a complex network, and introduces a novel community control strategy based on Liu's framework. First, the paper shows that it is nearly impossible to find an input sequence of signals to achieve the desired state because of control signal transmissions between target communities and the remainder network, especially without the topology of the entire network. Thus, the paper introduces a new kind of control nodes that block the influences of the remainder network on the control process of target communities, termed immune nodes. Four types of immune nodes are proposed and their merits and shortcomings are analyzed. Third, the paper proposes a strategy to efficiently control target communities without the entire topology of network. The strategy reduces the total number of driver nodes and control nodes by deliberately arranging the matching sequence of nodes. Finally, the paper carries out experiments on a series of model networks and 11 real networks. The experimental results show that the number of driver nodes and immune nodes required to control a community is less than the number of driver nodes needed to control the entire network, when the average degree of a network is low, but the ratio of immune nodes and driver nodes to all nodes increases. For example, when controlling the biggest community of the *celegansneural* network [11], the number of the driver nodes is reduced from 109 to 34 and the number of the immune nodes is 28, but the fraction of the driver nodes and immune nodes increased from 36.7% to 58.62%.

## 2. Analysis of community control of complex network

### 2.1. Complex network controllability based on the linear system control theory

This section first introduces the basic concepts and definitions of the complex network controllability model based on the linear system control theory [1]. A complex network is controllable if and only if it satisfies Kalman's controllability rank condition [12]. In controlling the states of all nodes in a complex network $G(V, E)$, the control node is a node into which control signals are inputted, denoted as $v_u$, and the control node set is denoted as $V_u$; the input node is an additional node that inputs control signals to control nodes; the driver nodes are the control nodes that do not share input nodes, denoted as $v_D$; the minimum driver node set (MDS) $V_D$ is the driver node set that can fully control the states of all nodes in the network and has the minimum number $|V_D|$ of driver nodes. If the state of a node $v_i$at time $t$ is denoted as $x_i(t)$, then it is possible to denote the state of a network as $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$. The equation of the state changing of the nodes in a network is:

$$x'(t) = Ax(t) + Bu(t) \tag{1}$$

where $A$ is the transpose of the adjacency matrix of the network, $B$ is the input matrix that shows how control signals are inputted, and $u(t) = (u^1(t), u^2(t), \ldots, u^M(t))^T$ is the state of $M$ input nodes at time $t$.

In order to obtain the minimum driver node set, Liu et al. [1] apply Lin's structural controllability theory [13] and derive the minimum input theorem [1] which shows that the MDS of a network can be derived by finding the maximum matching of its graph.

### 2.2. Influence of external connections upon community control

Although complete controllability of a target community can be fully achieved by determining its MDS, finding a sequence of inputs able to achieve a given control goal can be very difficult due to the connections between the target community and the remainder of the network. Because control signals inputted into a target community would be transmitted to and come back from the remainder network through these connections, the returned signals would influence the control process and a target community could not be driven to the planned states. Moreover, the influence would be unpredictable, if the topology besides target communities is unknown.

Figs. 1 and 2 show a simple example in which just one node outside a target community could greatly influence the control process of a community. In the network $G_1$ of Fig. 1 and $G_2$ of Fig. 2, the white node is an input node, $v_1^1$ of $G_1$ and $v_2^1$ of $G_2$ are driver nodes and two red nodes are members of the target community. The difference between $G_1$ and $G_2$ is