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Construction of a Langevin model from time series with a periodical correlation function: Application to wind speed data

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HIGHLIGHTS

- A Langevin-type equation for stochastic processes with a periodical correlation function is introduced.
- A procedure of reconstruction of the equation from time series is proposed and verified on simulated data.
- The method is applied to hourly time series of wind speed in order to construct a macroscopic model of the phenomenon.

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ABSTRACT

A Langevin-type equation for stochastic processes with a periodical correlation function is introduced. A procedure of reconstruction of the equation from time series is proposed and verified on simulated data. The method is applied to geophysical time series – hourly time series of wind speed measured in northern Italy – constructing the macroscopic model of the phenomenon.

1. Introduction

Geophysical processes are very complex; they are characterized by hidden initial and boundary conditions. For that reason, stochastic modeling is often the most adequate manner to describe them, where unknown elements of the process are treated as random with an appropriate distribution. As an example, crack evolution in earthquake processes was explained through some kinetic models derived following the kinetic theory of gases [1,2]. The models were applied to the problem of electric earthquake precursors [3–6]. Recently, non-extensive statistical physics has become an interesting approach, leading to many macroscopic relations in geophysics [7–10].

In order to explain some aspects of geophysical phenomena, cellular automata with stochastic mechanisms are useful. For instance, the time series of measurable quantities (observables) can be modeled by means of linear ARMA or even nonlinear Langevin equations, leading to a macroscopic description of the phenomenon under investigation. In Ref. [11,12], a link between this macroscopic description and the microscopic structure and rules of adequate cellular automata was proposed [13–15].

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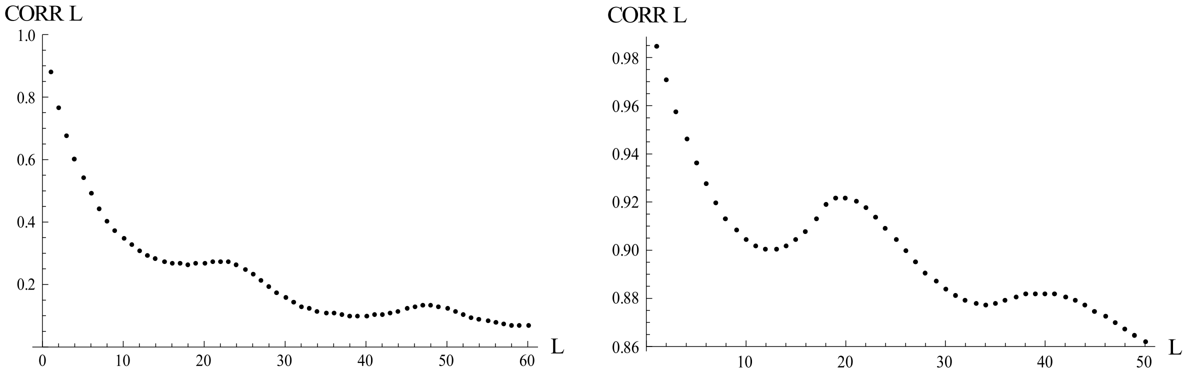

Fig. 1. Correlation function of (a) time series of the hourly speed of wind, (b) time series generated by the Langevin-type equation (3) with $a(y) = -0.5y$, $b(y) = 1$, $L = 20$, $d = 0.5$.

Linear stochastic ARMA models (see, e.g., Refs. [16,17]) are Markovian of arbitrary order p, which is usually greater than 1. The structure of these models can have a rather complicated correlation function. In contrast, Langevin equations describe Markov processes of only order 1, but since they can be nonlinear the distribution functions are non-Gaussian. The correlation function of the stochastic process generated by a Langevin equation is exponential. The Ito–Langevin equation has been applied to model many time series (see, e.g., Refs. [18–21]).

Sometimes in natural processes, although the time series do not show evident cyclic behavior, their correlation functions are characterized by periodicity; ARMA models are not appropriate, because it can be shown that in this case the distribution function has an exponential tail (for the identification of an exponential tail in critical phenomena see Ref. [22]); therefore, some stochastic nonlinear models should be applied.

This paper is organized as follows. In Section 2, a nonlinear Langevin-type equation with periodic behavior of correlations for the chosen lag will be presented; in Section 3, a method of reconstruction of such equations from time series is proposed, along with testing by means of simulated data; in Section 4, the method is applied to the observational time series of wind speed measured in northern Italy, already analyzed to extract their dynamical properties [23].

2. Langevin-type equation

The discrete Langevin equation has the form (see, e.g., [24])

$$y(t + \Delta t) = y(t) + a(y(t))\Delta t + \sqrt{b(y(t))}\sqrt{\Delta t}R_t, \tag{1}$$

where $a(y)$ corresponds to the deterministic force (drift), $b(y)$ to the stochastic force (diffusion), and R_t is an independent random variable with normal density.

We define the random function $c(\Delta^L y, R_t, d, r_t)$ as follows:

$$c(\Delta^L y, R_t, d, r_t) = \begin{cases} 1 & \text{if } (\Delta^L y < 0 \wedge R_t \geq 0) \vee (\Delta^L y < 0 \wedge R_t < 0 \wedge r_t > d) \vee (\Delta^L y \geq 0 \wedge R_t \geq 0 \wedge r_t \leq d) \\ -1 & \text{if } (\Delta^L y \geq 0 \wedge R_t < 0) \vee (\Delta^L y < 0 \wedge R_t < 0 \wedge r_t \leq d) \vee (\Delta^L y \geq 0 \wedge R_t \geq 0 \wedge r_t > d), \end{cases} \tag{2}$$

which depends on the sign of the difference $\Delta^L y(t) = y(t) - y(t - (L-1)\Delta t)$, the sign of the current random Wiener variable R_t, on an interdependence parameter d, and on a random variable r_t.

In order to take into account the periodic behavior of correlations for the lag L, by means of Eq. (2), we modify the Langevin equation as follows:

$$y(t + \Delta t) = y(t) + a(y(t))\Delta t + c(\Delta^L y, R_t, d, r_t)\sqrt{b(y(t))}\sqrt{\Delta t}|R_t|. \tag{3}$$

In Eq. (3) (called a Langevin-type equation hereafter), r_t is a random variable with uniform distribution in (0, 1), while d is a parameter in [0, 1]. When the difference $\Delta^L y(t)$ is negative, at step $(t + \Delta t)$, $c(\Delta^L y, R_t, d, r_t) = 1$ with higher probability for small d (and vice versa). In this way, the function $c(\Delta^L y, R_t, d, r_t)$ represents a relationship between the values of $y(t - (L-1)\Delta t)$ and those of $y(t + \Delta t)$, which are separated by the interval $L\Delta t$. For $d = 0$, the interrelation is maximal. The condition $d = 1$ leads to the original Langevin equation (Eq. (1)):

$$c(\Delta y, R_t, r_t) = \begin{cases} 1 & \text{if } R_t \geq 0 \\ -1 & \text{if } R_t < 0. \end{cases} \tag{4}$$

As an example, Fig. 1(b) shows the correlation function calculated from time series generated by the Langevin-type equation (Eq. (3)) with the simple linear function $a(y) = -0.5y$, constant $b(y) = 1$, and parameters $L = 20$, $d = 0.5$.