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Fundamental diagram in traffic flow of mixed vehicles on multi-lane highway

Katsunori Tanaka, Takashi Nagatani *, Shuichi Masukura

Department of Mechanical Engineering, Division of Thermal Science, Shizuoka University, Hamamatsu 432-8561, Japan

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ABSTRACT

We study the fundamental diagram for traffic flow of vehicular mixture on a multi-lane highway. We present the car-following model of multi-lane traffic in which slow and fast vehicles flow with changing lanes. We investigate the traffic states of the vehicular mixture under the periodic boundary. Two values of the current appear at a density and two current curves are obtained. Vehicles move with changing lanes in the traffic state of high current, while vehicles move without changing lanes in the traffic state of low current. They depend on the density, the fraction of slow vehicles, and the initial condition. In the high-current curve, the jamming transition between the free flow and the jammed state occurs at a low density. The fundamental diagrams (current–density diagrams) are shown for the single-lane, two-lane, three-lane, and four-lane traffics.

1. Introduction

Mobility is nowadays one of the most significant ingredients of a modern society. Traffic flow is a kind of self-driven many particle system [1–5]. Vehicular traffic has been studied by several traffic models: car-following models, cellular automaton (CA) models, gas kinetic models, and hydrodynamic models [6–34]. When traffic networks in a city often exceed the capacity, jams occur. Traffic jams are typical signature of the complex behavior of traffic flow.

Also, in some emergency events, people often evacuate from the city by means of vehicles. Traffic modeling is important in Hurricane evacuation [29,30]. Until now, a single-lane traffic flow has been investigated mainly. Such traffic states as the stop- and go-waves have been found. However, the traffic simulation for the single-lane highway is not sufficient because vehicles cannot pass other vehicles. In real traffic, a highway has multi-lanes from two lanes to four lanes. Also, slow and fast vehicles are mixed on a multi-lane highway. Wager, Nagel, and Wolf have presented the CA model for multi-lane traffic [35]. Chowdhury, Wolf, and Schreckenberg have proposed the particle hopping model for two-lane traffic with two kinds of vehicles [36]. Knospe et al. have studied the disorder effect in a two-lane CA model [37]. Honk effect, aggressive lane changing behavior, the lane reduction effect, and metastability have been studied in the two-lane CA model [38–41]. The CA models have been used to investigate the traffic behavior on a two-lane highway. However, the traffic flow of mixed vehicles on the multi-lane highway has been little studied until now from the point of view of the optimal-velocity models described by the differential equations. The optimal-velocity model has such an advantage that position and time are not discrete but have a real value. The lane changing is performed in parallel update in the CA model, while it is carried out in shuffled update in the optimal-velocity model. In the CA model for multi-lane traffic, the lane changing rules are not simple

* Corresponding author.
E-mail address: tmtnaga@ipc.shizuoka.ac.jp (T. Nagatani).

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because it might happen that two vehicles change to the same position at the same time. The lane changing rules are simple in the multi-lane optimal-velocity model because one uses the shuffled update.

In this paper, we study the traffic flow of mixed vehicles on the multi-lane highway where vehicles pass each other by lane changing. Especially, we investigate the traffic flow at high sensitivity where the stop- and go-wave does not occur because one is interested in the traffic states induced by both effects of slow vehicles and multi-lanes and uses the analytical results (theoretical current curves). We study the jamming transition induced by the competition between the blocking of slow vehicles and passing of fast vehicles. We present a car-following model of multi-lane traffic in which slow and fast vehicles move with changing lanes. We derive the fundamental diagrams for the multi-lane traffic numerically. We clarify the traffic states and the jamming transition.

2. Model and theoretical current curves

We consider a situation where many vehicles move ahead with changing lane on a multi-lane highway. Two kinds of vehicles with low and high velocities are introduced on the multi-lane highway. Traffic flow is under the periodic boundary condition. We assume that fast vehicles pass over the other vehicles by changing lane if the criteria of lane changing are satisfied.

If there are no specific public rules, it is natural to consider symmetric incentive criteria. We adopt symmetric lane changing rules. Lane changing is implemented as a pure sideways movement. We assume that the vehicular movement is divided into two parts: one is the forward movement and the other is the sideways movement. We apply the optimal-velocity model to the forward movement [1,28]. The optimal-velocity model is described by the following equation of motion of vehicle i:

$$\frac{\mathrm{d}^2 x_i}{\mathrm{d}t^2} = a\left\{V(\Delta x_i) - \frac{\mathrm{d}x_i}{\mathrm{d}t}\right\}, \tag{1}$$

where $V(\Delta x_i)$ is the optimal-velocity function, $x_i(t)$ is the position of vehicle i at time t, $\Delta x_i(t)(= x_{i-1}(t) - x_i(t))$ is the headway of vehicle i at time t, and a is the sensitivity (the inverse of the delay time) [1].

A driver adjusts the vehicular speed to approach the optimal velocity determined by the observed headway. The sensitivity a allows for the time lag $\tau = 1/a$ that it takes the vehicular speed to reach the optimal velocity when the traffic is varying. Generally, it is necessary that the optimal-velocity function has the following properties: it is a monotonically increasing function and it has an upper bound (maximal velocity). The optimal-velocity function of fast vehicles is given by

$$V_f(\Delta x_i) = \frac{v_{f,\max}}{2}\left[\tanh(\Delta x_i - x_c) + \tanh(x_c)\right], \tag{2}$$

where $v_{f,\max}$ is the maximal velocity of fast vehicles and x_c the position of turning point [1]. x_c means the safety distance [1].

The optimal-velocity function of the slow vehicle is given by

$$V_s(\Delta x_i) = \frac{v_{s,\max}}{2}\left[\tanh(\Delta x_i - x_c) + \tanh(x_c)\right], \tag{3}$$

where $v_{s,\max}$ is the maximal velocity of the slow vehicle and $v_{s,\max} < v_{f,\max}$.

We adopt the following lane changing rule for the two-lane highway:

$$\begin{aligned} &\Delta x_i < 2x_c \quad \text{for the incentive criterion,} \\ &\Delta x_{fi} > \Delta x_i \quad \text{and} \quad \Delta x_{bi} > x_c \quad \text{for the security criterion,} \end{aligned} \tag{4}$$

where Δx_{fi} is the headway between vehicle i and the vehicle ahead on the target lane and Δx_{bi} is the headway between vehicle i and the vehicle behind on the target lane. Fig. 1 shows the schematic illustration of lane changing for the two-lane highway. A driver wants to change the lane when the headway is less than two times the safety distance. In addition to the incentive criterion, when the headway between his vehicle and the front vehicle on the target lane is larger than his headway and the headway between his vehicle and the back vehicle on the target lane is larger than the safety distance, it is successful for his vehicle to change the lane.

We adopt the following lane changing rule except for the edge lanes for the multi-lane highway higher than two lanes:

$$\begin{aligned} &\Delta x_i < 2x_c \quad \text{for the incentive criterion,} \\ &\Delta x_{fli} > \Delta x_i \quad \text{and} \quad \Delta x_{bli} > x_c \quad \text{for the security criterion 1,} \\ &\Delta x_{fri} > \Delta x_i \quad \text{and} \quad \Delta x_{bri} > x_c \quad \text{for the security criterion 2,} \end{aligned} \tag{5}$$

where Δx_{fli} (Δx_{fri}) is the headway between vehicle i and the vehicle ahead on the left (right) target lane and Δx_{bli} (Δx_{bri}) is the headway between vehicle i and the vehicle behind on the left (right) target lane. Fig. 2 shows the schematic illustration of lane changing on a lane except for the edge lanes for the multi-lane highway. A driver wants to change the lane when the headway is less than two times safety distance. In addition to the incentive criterion, when the headway between his vehicle and the front vehicle on the target lane is larger than his headway and the headway between his vehicle and the back