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Adaptive synchronization between two complex networks with nonidentical topological structures

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ABSTRACT

This paper addresses the theoretical analysis of synchronization between two complex networks with nonidentical topological structures. By designing effective adaptive controllers, we achieve synchronization between two complex networks. Both the cases of identical and nonidentical network topological structures are considered and several useful criteria for synchronization are given. Illustrative examples are presented to demonstrate the application of the theoretical results.

1. Introduction

The structure of many real systems in nature can be described by complex networks. Complex networks with the well known small-world (SW) property [1] and scale-free (SF) property [2] such as the human nervous systems, social groups, world wide web and metabolic networks [3] have attracted a lot of attention from researchers across a variety of disciplines including physics, sociology, biology, mathematics, engineering science and so on.

On the basis of complex network models, investigation of the properties of complex systems has been reported recently. In particular, the synchronous problem of networks is a main focus in this research gold rush. Synchronization in networks is a very common phenomenon in real systems and it is manifested by the appearance of some forms of relations between the functionals of different dynamical variables as a result of interactions. Pecora and Carrol [4] applied the Master Stability function approach to the study of stability of synchronization in linearly coupled networks. Wu et al. [5,6] investigated synchronization in an array of linearly coupled dynamical systems by Lyapunov's direct method and proved that strong enough mutual diffusive coupling will synchronize an array of identical cells. Also, Wang et al. [7,8] gave a detailed description of the synchronization of specific kinds of complex networks such as small-world networks and scale-free networks.

The synchronization in a network as mentioned above is called "inner synchronization" in Ref. [9], as it is a collective behavior within a network. Moreover, one may also consider "outer synchronization", which refers to the synchronization phenomenon between two or more complex networks regardless of synchronization of the inner network. Recently,

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the synchronization between two complex dynamical networks was studied by Li, Sun and Kurths [9]. Using an open-plus-closed-loop controller, they realized the synchronization between two networks having the same topological structure. However, detailed analysis of the synchronization between two networks of different topological structures has not been attempted. In this paper, we propose a new method to tackle the analysis of synchronization between two networks.

A focused problem in network control and synchronization is related to the design of physically applicable controllers to effectively stabilize or synchronize coupled dynamical systems. In Refs. [10–13], the authors use adaptive-feedback controllers to achieve synchronization of chaotic systems or networks. In this paper, we will prove that the synchronization between two complex dynamical networks with identical topologies and nonidentical topologies can be achieved by designing appropriate adaptive controllers. We will also show that the adaptive control method can be applied to achieve synchronization between networks of different topological structures such as star-shape, ring-shape, non-diffusion coupled network structures, and so on.

This paper is organized as follows. In Section 2, we introduce the model for studying coupled networks and theoretically analyze the synchronization of two networks with identical and nonidentical coupling configurations under the application of adaptive controllers. Several numerical examples for illustrating synchronization of two networks with identical and nonidentical topological structures are given in Section 3. The conclusion will be given in Section 4.

2. Network model and preliminaries

2.1. Network model

Consider a dynamical network consisting of N linearly coupled identical nodes, with each node being an n-dimensional dynamical systems described by

$$\dot{x}_i(t) = f(x_i(t)) + \sum_{j=1}^{N} c_{ij}\,\mathbf{A}x_j(t), \quad i = 1, 2, \ldots, N \tag{1}$$

where $x_i = (x_{i1}, x_{i2}, \ldots, x_{in},)^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector of the ith node, $f : \mathbb{R}^n \to \mathbb{R}^n$ is a smooth nonlinear vector field, the node dynamical function is $\dot{x} = f(x)$, $\mathbf{A} \in \mathbb{R}^{n\times n}$ is a constant matrix linking the coupled variables, and $\mathbf{C} = (c_{ij})_{N\times N}$ is the coupling configuration matrix representing the coupling strength and the topological structure of the network, in which c_{ij} is non-zero if there is a connection from node i to node j ($i \neq j$), and is zero otherwise.

Here, the configuration matrix $\mathbf{C}$ need not be symmetric or irreducible, and the diffusive couplings are $c_{ii} = -\sum_{j=1,j\neq i}^{N} c_{ij},\ i = 1, 2, \ldots, N$. Also the inner coupling matrix $\mathbf{A}$ is not necessarily symmetric. The simple uniform dynamical network [8,7],

$$\dot{x}_i(t) = f(x_i(t)) + \epsilon \sum_{j=1}^{N} c_{ij}\,\mathbf{A}x_j(t), \quad i = 1, 2, \ldots, N \tag{2}$$

where ϵ is the coupling strength and coupling matrix $\mathbf{C}$ is a 0-1 matrix, is a special case of the network given by (1).

We take the network given by (1) as the driving network, and the response network with an adaptive control scheme which is given by

$$\dot{y}_i(t) = f(y_i(t)) + \sum_{j=1}^{N} d_{ij}\,\mathbf{A}y_j(t) + u_i, \quad i = 1, 2, \ldots, N \tag{3}$$

where y_i, N, f, and $\mathbf{A}$ have the same meanings as those in (1), $\mathbf{D} = (d_{ij})_{N\times N}$ is the coupling configuration matrix, and u_i is the controller for node i to be designed according to the specific network structures $\mathbf{C}$ and $\mathbf{D}$.

2.2. Preliminaries

In this subsection, a useful definition and an assumption, which are relevant to the subsequent study, are given.

Definition 1. Let $x_i(t, \mathbf{X}_0)$ and $y_i(t, \mathbf{Y}_0, u_i)$ ($i = 1, 2, \ldots, N$) be the solutions of the network given by (1) and (3), where $\mathbf{X}_0 = (x_1^0, x_2^0, \ldots, x_N^0)^{\mathrm{T}}$, $\mathbf{Y}_0 = (y_1^0, y_2^0, \ldots, y_N^0)^{\mathrm{T}} \in \mathbb{R}^{nN}$, and $f : D \to \mathbb{R}^n$ are the continuously differentiable mappings with $D \subseteq \mathbb{R}^n$, for all t. If there is a nonempty open subset $E \subseteq D$, with $x_i^0, y_i^0 \in E$, such that $x_i(t, X_0), y_i(t, Y_0) \in D$ for all $t \geq 0, i = 1, 2, \ldots, N$, and

$$\lim_{t\to\infty} \|y_i(t, \mathbf{Y}_0, u_i) - x_i(t, \mathbf{X}_0)\| = 0, \quad \text{for } 1 \leq i \leq N, \tag{4}$$

then the dynamical networks (1) and (3) are said to be synchronized.