Physica A 392 (2013) 3223–3230

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Effect of restart at signals on traffic flow through a series of signals

Yuki Hino, Kazuhiro Tobita, Takashi Nagatani *

Department of Mechanical Engineering, Division of Thermal Science, Shizuoka University, Hamamatsu 432-8561, Japan

HIGHLIGHTS

- We studied the effect of restart at signals on the traffic flow controlled by a series of signals.
- We presented the difference equation for describing the traffic flow of the NS model with signals.
- We derived the dependence of fundamental diagram on the restart effect and signal's characteristic.

ARTICLE INFO

Article history:
Received 11 October 2012
Available online 2 April 2013

Keywords:
Traffic dynamics
Signal control
Restart effect
Complex system

ABSTRACT

We study the effect of restart at signals on the vehicular traffic controlled by a series of signals. The Nagel–Schreckenberg model (NS model) and Fukui–Ishibashi model (FI model) are applied to the vehicular motion. In the FI model, the step-by-step acceleration is not taken into account but the acceleration effect is included in the NS model. It is shown that the difference between both models results in the restart effect at signals. The extended version of the NS model with signals is formulated by the difference equation. The restart at signals has an effective effect on the traffic flow. The fundamental diagram changes highly by the restart effect. The dependences of mean speed on the cycle time are shown.

1. Introduction

Mobility is nowadays one of the most significant ingredients of a modern society. Recently, physicists have been interested in transportation problems [1–5]. The concepts and techniques of physics are being applied to transportation systems. The traffic dynamics has been studied from a point of view of statistical mechanics [6–15].

In city traffic network, there are many signals. Vehicles and walkers are controlled by traffic lights (signals) to give priority for a road and to ensure road safety because danger is present at crossings. In real traffic, the vehicular traffic depends highly on the signal's characteristics and the control strategy. Brockfeld et al. have studied optimizing traffic lights for city traffic by using a NS model [16]. Sasaki and Nagatani have investigated the traffic flow controlled by signals on a single-lane roadway by using the optimal velocity model [17].

Until now, the periodic traffic controlled by a few traffic lights has been studied [16,17]. Recently, a few works have been carried out for the traffic of vehicles moving through an infinite series of traffic lights. The effect of a signal's characteristic on vehicular traffic has been studied [18–24]. Lammer and Helbing have studied the effect of self-controlled signals on vehicular flow based on fluid-dynamic and many-particle simulations [25].

The influence of dynamic information on the traffic flow has been investigated using a route choice scenario [26–29]. In the two-route traffic systems studied until now, there were no traffic signals. Very recently, the two-route traffic problem

* Corresponding author. Tel.: +81 53 478 1048; fax: +81 53 478 1048.
E-mail addresses: tmtnaga@ipc.shizuoka.ac.jp, wadokeioru@yahoo.co.jp (T. Nagatani).

0378-4371/$ – see front matter
http://dx.doi.org/10.1016/j.physa.2013.03.027

with no signals has been extended to that with signals. It has been shown that the route choice depends highly on the signal's characteristics [30].

Traffic flow through the series of signals has been studied, mainly, by the use of CA models and the car-following models (optimal velocity model). In particular, the NS model and FI model have been applied to city traffic with signals. In the traffic flow with no signals, the fundamental diagram obtained by the NS model agrees with that obtained by the FI model. However, the vehicular motion in the NS model is definitely different from that in the FI model. In the FI model, a vehicle accelerates instantly until the maximum speed if there is a sufficiently large headway, while a vehicle accelerates step by step to the maximum speed in the NS model. The restart motion at signals is not taken into account in the FI model but it is included in the NS model. There is a difference between both models for the restart motion at signals. It is important to know how the restart motion affects the traffic flow and fundamental diagram. However, the restart motion at signals has not been studied until now.

Also, the extended NS model including signals has been simulated by the use of the update rule. The dynamics has not been described in terms of the difference equation. The difference equation model is more suitable to the computer simulation because the dynamics is described by the equation.

In this paper, we study the effect of the restart motion at signals on the traffic behavior. We present the formulation of the difference equation for the extended NS model including signals. We compare the traffic characteristic of the FI model with that of the NS model. We derive the fundamental diagrams for both models. We show the dependence of the fundamental diagram (current–density diagram) on the signal characteristic. Also, we derive the dependence of the mean velocity on the signal's characteristic. We clarify the effect of restart motion at signals on the traffic behavior.

2. NS and FI models and difference equations for traffic flow through signals

We consider the flow of vehicles going through the series of traffic lights on a one-dimensional lattice. Each vehicle does not pass over other vehicles. The traffic lights are positioned homogeneously on a roadway. The interval between signals has a constant value and is given by l. All traffic lights change from red (green) to green (red) with a fixed time period $(1 - s_p)t_s$ (s_pt_s). The period of green is s_pt_s and the period of red is $(1 - s_p)t_s$. Time t_s is called the cycle time and fraction s_p represents the split which indicates the ratio of green time to cycle time.

We apply the FI and NS models to the vehicular motion [1,6,31]. We define the position of vehicle i at time t as $x_i(t)$ where x, i, and t are an integer. The velocity takes the integer value ranging from 0 to $v_{\max}$. The dynamics of the FI model is formulated as follows:

$$x_i(t+1) = x_i(t) + \min[v_{\max}, x_{i+1}(t) - x_i(t) - 1], \tag{1}$$

where $\min[A, B]$ is a minimum function and takes the minimum value within A and B. If headway $\Delta x_i(t) (= x_{i+1}(t) - x_i(t))$ is larger than the maximum velocity, the vehicle moves with the maximum velocity. If the headway is less than the maximum velocity, the vehicle moves with velocity $\Delta x_i(t) - 1$. The velocity at time t is given by $\min[v_{\max}, \Delta x_i(t) - 1]$. When a vehicle restarts from the stationary state, the velocity increases instantly to $\min[v_{\max}, \Delta x_i(t) - 1]$. In the FI model, the delay of acceleration is not taken into account. Eq. (1) is the difference equation for FI model with no signals.

In the NS model, the motion of vehicles is described by the following rules:

Rule 1. Acceleration: $v_i \leftarrow \min(v_i + 1, v_{\max})$

Rule 2. Deceleration: $v_i' \leftarrow \min(v_i, gap)$

Rule 3. Randomization: with a certain probability p do $v_i'' \leftarrow \max(v_i' - 1, 0)$

Rule 4. Movement: $x_i \leftarrow x_i + v_i''$.

Vehicles are updated in parallel according to four steps: acceleration, deceleration, randomization, and movement. The dynamics of the NS model is formulated as follows:

$$x_i(t+1) = x_i(t) + \max[0, \min\{v_{\max}, x_{i+1}(t) - x_i(t) - 1, x_i(t) - x_i(t-1) + 1\} - \xi_i(t)], \tag{2}$$

where the Boolean random variable $\xi_i(t) = 1$ with probability p and 0 with probability $1 - p$. Rule 1 describes the step-by-step acceleration. The acceleration is represented by $\min\{v_{\max}, x_{i+1}(t) - x_i(t) - 1, x_i(t) - x_i(t-1) + 1\}$. When the probability of the random deceleration is zero, the main difference between FI and NS models is the acceleration. In the NS model, when a vehicle restarts from the stationary state, the velocity increases step by step from zero to $\Delta x_i(t) - 1$. Eq. (2) is the difference equation for the NS model with no signals.

We extend the difference equations (1) and (2) of FI and NS models to take into account traffic lights. When a vehicle arrives at a traffic light and the traffic light is red, the vehicle stops at the position of the traffic light. Then, when the traffic light changes from red to green, the vehicle goes ahead. On the other hand, when a vehicle arrives at a traffic light and the traffic light is green, the vehicle does not stop and goes ahead without changing speed. The position of the closest signal before vehicle i at time t is given by

$$x_{i,s}(t) = \left\{ \operatorname{int}\left(\frac{x_i(t)}{l}\right) \right\} l. \tag{3}$$