Physica A 387 (2008) 5012–5016

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Numerical tests of a new molecule-dependent momentum transport equation

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ARTICLE INFO

Article history:
Received 6 March 2008
Received in revised form 29 April 2008
Available online 9 May 2008

PACS:
0.5.20.Jj
05.60.Cd
47.11.Bc
47.27.Cn
47.27.ek

Keywords:
Momentum transport
Onset of turbulence
Poiseuille flow

ABSTRACT

It is shown that a new incompressible fluid equation is obtained by inclusion of a new dimensionless coupling parameter in the momentum transport equation derived in [L. Jirkovsky, L. Bo-ot, Momentum transport equation for the fluids using projection–perturbation formalism and onset of turbulence, Physica A 352 (2005) 241–251] from the Boltzmann kinetic equation where the Boltzmann collision integral includes inelastic interactions of quantum origin among the particles of the fluid. Numerical results from the equation for the plane and circular Poiseuille flows are consistent with the observations. The numerical tests also manifest a difference in the onset of turbulence between the flat plates and the circular pipe flow systems. Although all obtained velocity profiles are flattened at the center – a feature of turbulence – the results demonstrate greater stability of the velocity profiles in the circular pipe flow.

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1. Introduction

In a paper on laminar–turbulent transition [1], second- and third-order momentum transport equations were derived using projection–perturbation formalism. As a result a molecular parameter of quantum origin naturally appeared in the equations. In this paper we use the new incompressible momentum transport equation obtained by the inclusion of a new coupling parameter as a better approximation in representing higher-order momentum transport equations. It is necessary that this equation be tested numerically. We report the first computational tests of such an approach. We apply flat plate and circular pipe boundary conditions to generate the respective velocity fields. We explore and compare the generated velocity profiles in both geometries to see the difference in turbulent behavior and onset of turbulence.

It is well established that the velocity profile of turbulent flow between two flat plates is parabolic and flattened towards the center while for turbulent flow in circular pipe the velocity profile forms a surface of a classic flattened paraboloid [2,3]. This cannot be explained by the standard incompressible Navier–Stokes equation because molecular structure is ignored and hence difficulties in explaining the laminar–turbulent transition and origin of turbulence arise.

The molecular parameter in our study represents the strength of inelastic interactions related to the internal structure of the fluid molecules and is explained if one adopts the hypothesis of a microscopic origin of turbulence [4–7]. Inelastic interactions among the molecules of the fluid can be due to excitations of molecules to higher rotational levels resulting in

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doi:10.1016/j.physa.2008.05.006

loss of momentum and kinetic energy of the particles as explained in Refs. [5–7]. Turbulence is known to be a dissipative process. Monatomic fluid has been modeled as a quantum confinement with discrete energy levels. The quantum kinetic model of turbulence [7] is based on the idea of associating turbulence with deterministic chaos [4,8].

Although Landau regarded turbulence as a classical problem originating from the nonlinearity of the Navier–Stokes equation, as a precursor to the theory of deterministic chaos, Landau in 1944 considered turbulence in time as a limit of a sequence of an infinite number of instabilities each creating a new basic frequency. However in 1971 it was shown by Ruelle and Takens and later in 1978 by Newhouse, that three instabilities creating three incommensurable frequencies could already induce chaotic motion or turbulence. Turbulence in time is interpreted as the onset of turbulence [8].

Experimental evidence have been reported by Nerushev, Novopashin and Muriel suggesting the existence of slight differences in the critical Reynolds numbers for different gases as predicted by the theory, particularly for carbon monoxide and nitrogen [9–11]. The difference is a consequence of different energy gaps between two rotational levels of the molecules.

The results reported here such as the flattened velocity profiles, the difference in laminar–turbulent transition and the greater stability of the flow in a circular pipe apparently reflect the presence of the molecular parameter in the new momentum transport equation.

2. New momentum transport equation

Based on the second- and third-order fluid equations with molecular parameter b representing inelastic interactions among the molecules of the fluid derived in Ref. [1] from the Boltzmann kinetic equation using projection–perturbation formalism, we adopt an incompressible momentum transport equation of the form

$$\rho\left[\frac{\partial\vec{U}}{\partial t}+(\vec{U}\cdot\vec{\nabla})\vec{U}\right]+\vec{\nabla}p-\nu\rho\vec{\nabla}^2\vec{U}=\frac{b}{m}\int_0^t(t-s)^2\vec{\nabla}^2\vec{U}\mathrm{d}s-\frac{4\varepsilon b}{m^2}\int_0^t\vec{\nabla}^3\vec{U}^2\mathrm{d}s \tag{1}$$

with $\varepsilon\in\langle 0,1\rangle$ as a new dimensionless parameter. Eq. (1) reduces to the second-order equation for $\varepsilon=0$ and reduces to the third-order equation for $\varepsilon=1$. For a value of ε between 0 and 1, Eq. (1) is a good approximate of the higher-order equations. A justification comes from besides its utility, also from the fact that the Taylor expansion of the propagator G in Ref. [1] is an alternating series. As a result the time development of the velocity profiles of the flow are oscillating around the solution of the exact momentum transport equation as one raises the order of the approximate equation. The oscillations would diminish with raising the order. Specifically for the time development of the velocity profiles in the Poiseuille flow, mean velocities are decreasing in even orders while increasing in odd orders in time, as can be seen in numerical solutions of the second- and third-order equation.

3. Application to plane and circular Poiseuille flow and numerical tests

If one differentiates Eq. (1) four times with respect to time, it reduces to a fifth-order nonlinear equation for the flow between two flat plates with distance L apart, and with configuration where $\vec{U}=(U(z,t),0,0)$

$$\frac{1}{\nu}\frac{\partial^5U}{\partial t^5}-\frac{\partial^6U}{\partial t^4z^2}-\frac{2b}{m\nu}\frac{\partial^3U}{\partial t\partial z^2}+\varepsilon\frac{4!\,2b}{m^2\nu}\left(3\frac{\partial U}{\partial z}\frac{\partial^2U}{\partial z^2}+U\frac{\partial^3U}{\partial z^3}\right)=0. \tag{2}$$

Repeating the procedure for the flow in circular pipe of radius R with the configuration where $\vec{U}=(0,0,U(r,t))$, in cylindrical coordinates we have

$$\begin{aligned}&\frac{1}{\nu}r^2\frac{\partial^5U}{\partial t^5}-r^2\frac{\partial^6U}{\partial r^2\partial t^4}-r\frac{\partial^5U}{\partial r\partial t^4}-\frac{2b}{m\nu}\left(r^2\frac{\partial^3U}{\partial r^2\partial t}+r\frac{\partial^2U}{\partial r\partial t}\right)\\&\quad+\varepsilon\frac{4!2b}{m^2\nu}\left[3r^2\frac{\partial U}{\partial r}\frac{\partial^2U}{\partial r^2}+3r\left(\frac{\partial U}{\partial r}\right)^2+r^2U\frac{\partial^3U}{\partial r^3}+rU\frac{\partial^2U}{\partial r^2}\right]=0.\end{aligned} \tag{3}$$

3.1. Second-order equations ($\varepsilon=0$)

Eq. (2) for the plane Poiseuille flow reduces to

$$\frac{1}{\nu}\frac{\partial^4U}{\partial t^4}-\frac{\partial^5U}{\partial t^3z^2}-\frac{2b}{m\nu}\frac{\partial^2U}{\partial z^2}=0 \tag{4}$$

while Eq. (3) for the flow in circular pipe reduces to

$$\frac{1}{\nu}r^2\frac{\partial^4U}{\partial t^4}-r^2\frac{\partial^5U}{\partial r^2\partial t^3}-r\frac{\partial^4U}{\partial r\partial t^3}-\frac{2b}{m\nu}\left(r^2\frac{\partial^2U}{\partial r^2}+r\frac{\partial U}{\partial r}\right)=0. \tag{5}$$

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