Physica A 387 (2008) 5239–5245

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Two velocity difference model for a car following theory

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ARTICLE INFO

Article history:
Received 4 January 2008
Available online 15 March 2008

PACS:
05.70.Fh
05.70.Jk

Keywords:
Traffic flow
Car-following models
KdV soliton and kink–antikink soliton
Navigation

ABSTRACT

In the light of the optimal velocity model, a two velocity difference model for a car-following theory is put forward considering navigation in modern traffic. To our knowledge, the model is an improvement over the previous ones theoretically, because it considers more aspects in the car-following process than others. Then we investigate the property of the model using linear and nonlinear analyses. The Korteweg–de Vries equation (for short, the KdV equation) near the neutral stability line and the modified Korteweg–de Vries equation (for short, the mKdV equation) around the critical point are derived by applying the reductive perturbation method. The traffic jam could be thus described by the KdV soliton and the kink–anti-kink soliton for the KdV equation and mKdV equation, respectively. Numerical simulations are made to verify the model, and good results are obtained with the new model.

1. Introduction

Car-following models were proposed for the description of interacting driver–vehicle units. They have not only been of great importance with regard to an autonomous cruise control system, but also as important evaluation tools for intelligent transportation system strategies since the 1990s. And car-following theories have received much research interest. They are based on the assumption that each driver reacts in some specific fashion to a stimulus from the vehicle ahead of him.

In 1995, Bando et al. proposed a very attractive microscopic traffic model called the optimal velocity model (for short, OVM) [1]. It was based on the idea that each vehicle has an optimal velocity, which depends on the following distance of the preceding vehicle. It is based on the acceleration equation

$$\frac{\mathrm{d}^2 x_j(t)}{\mathrm{d}t^2} = a\left[V(\Delta x_j(t)) - \frac{\mathrm{d}x_j(t)}{\mathrm{d}t}\right], \tag{1}$$

where $x_j(t)$ is the position of car j at time t, $\Delta x_j(t) \equiv x_{j+1}(t) - x_j(t)$ is the headway between car j and car $j+1$ at time t, a is the sensitivity of a driver, and V is the optimal velocity function. Despite its simplicity and its few parameters, the OVM can describe many properties of real traffic flows, such as the instability of traffic flow, the evolution of traffic congestion, and the formation of stop-and-go waves. Helbing and Tilch [2] carried out a calibration of the OVM with respect to the empirical data. They adopted the following optimal velocity function

$$V(\Delta x_j(t)) = V_1 + V_2 \tanh[C_1(\Delta x_j(t) - l_c) - C_2], \tag{2}$$

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0378-4371/$ – see front matter © 2008 Published by Elsevier B.V.
doi:10.1016/j.physa.2008.02.081

Table 1
δt and c_j in different models

Model	δt (s)	c_j (km/h)
OVM ($a = 0.85\ \text{s}^{-1}$, $\lambda = 0\ \text{s}^{-1}$)	1.6	16.65
GFM ($a = 0.41\ \text{s}^{-1}$, $\lambda = 0.5\ \text{s}^{-1}$)	2.2	12.11
FVDM ($a = 0.41\ \text{s}^{-1}$, $\lambda = 0.5\ \text{s}^{-1}$)	1.4	19.03
TVDM ($a = 0.41\ \text{s}^{-1}$, $\lambda = 0.5\ \text{s}^{-1}$)	1.5	17.76

where $l_c = 5$ m is the length of the vehicles. The parameter values are

$$a = 0.85\ \text{s}^{-1}, \qquad V_1 = 6.75\ \text{m/s}, \qquad V_2 = 7.91\ \text{m/s}, \qquad C_1 = 0.13\ \text{m}^{-1}, \qquad C_2 = 1.57.$$

The comparison with the empirical data shows that OVM encountered the problems of too high acceleration and unrealistic deceleration. So Helbing and Tilch presented a generalized force model (for short, GFM) [2] to solve this problem. The governing equation is

$$\frac{\mathrm{d}^2 x_j(t)}{\mathrm{d}t^2} = a\left[V(\Delta x_j(t)) - \frac{\mathrm{d}x_j(t)}{\mathrm{d}t}\right] + \lambda\Theta(-\Delta v)(\Delta v), \tag{3}$$

where Θ is the Heaviside function, λ is a sensitivity coefficient different from a. But GFM cannot describe the delay time δt and the kinematic wave speed at jam density c_j properly (see Table 1). After that, Jiang and Wu put forward the full velocity difference model (for short, FVDM) [3]. The formula of FVDM reads

$$\frac{\mathrm{d}^2 x_j(t)}{\mathrm{d}t^2} = a\left[V(\Delta x_j(t)) - \frac{\mathrm{d}x_j(t)}{\mathrm{d}t}\right] + \lambda\Delta v. \tag{4}$$

Since empirical deceleration and acceleration are limited between the region $[-3\ \text{m/s}^2, 4\ \text{m/s}^2]$ [2], the FVDM has too high deceleration (see Fig. 3).

Recently, based on applying the Intelligent Transportation System (for short, ITS), cooperative driving models related to microscopic car-following model and macroscopic lattice hydrodynamic model were investigated by the authors [4,5], where ITS application meant that drivers could receive information of other vehicles on roads, and then adjust the velocities of their own vehicles. In light of this information, it is possible to improve the stability of traffic flow and suppress the appearance of traffic jams. In addition the KdV soliton and the kink–antikink soliton appearing as traffic jams in distinct regions such as a metastable region and an unstable region have been studied by a few researchers [4–9], these solitons correspond to the solutions of KdV and mKdV equations respectively.

In order to improve the OVM and considering the ITS application, we put forward a new model taking into account the velocity difference Δv_n and Δv_{n+1}, where $\Delta v_n \equiv v_{n+1} - v_n$. The linear analysis is conducted. We investigated the density waves in the metastable and unstable regions and derived the KdV and mKdV equations. By using the conclusion in paper [10], we obtained the KdV and kink–antikink soliton solutions quickly.

2. Model

With the rapid development of modern traffic, ITS plays an important role. By using such navigation, drivers can obtain the information that they need. In accordance with the above concept, on the basis of OVM, taking both Δv_n and Δv_{n+1} into account, we obtain a more useful model called the two velocity difference model (for short, TVDM), one whose dynamics equation is

$$\frac{\mathrm{d}^2 x_j(t)}{\mathrm{d}t^2} = a\left[V(\Delta x_j(t)) - \frac{\mathrm{d}x_j(t)}{\mathrm{d}t}\right] + \lambda G(\Delta v_n, \Delta v_{n+1}), \tag{5}$$

we select the parameters as that of GFM. $G(\cdot)$ is a generic, monotonically increasing function, and we assume a linear form as

$$G(\Delta v_n, \Delta v_{n+1}) = p\Delta v_n + (1-p)\Delta v_{n+1}, \tag{6}$$

where p is the weighting value. In the later simulation, we select $p = 0.86$, for we know that the influence of the vehicle ahead on the vehicle motion reduces gradually as the distance between the considered vehicle and the vehicle ahead increases. Also, the proper value of p could lead to desirable results.

Now, we apply TVDM to simulate the car motion under a traffic signal and examine certain properties of TVDM. At first, a traffic signal is red and all cars are waiting with a headway of 7.4 m, at which the optimal velocity is zero. Then at time $t = 0$, the signal changes to green and cars start. We define the delay time of car motion by δt as that in FVDM, which is related to the weighting value p. Then, we can estimate the kinematic wave speed at jam density $c_j = 7.4/\delta t$. For comparison, we use the same parameters as those in FVDM. The simulation results are shown in Fig. 1 and Table 1.

From Table 1, we can see that the observed δt is of the order of 1 s, just as Bando et al. [11] pointed out and c_j ranges between [17 km/h, 23 km/h] [12]. Therefore, TVDM is successful in anticipating the two parameters.