Physica A 387 (2008) 5263–5270

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Precursory phenomena associated with large avalanches in the long-range connective sandpile (LRCS) model

Ya-Ting Lee [a], Chien-chih Chen [a,*], Young-Fo Chang [b], Ling-Yun Chiao [c]

[a] Graduate Institute of Geophysics, National Central University, Jhongli, 320, Taiwan, ROC

[b] Institute of Seismology, National Chung-Cheng University, Ming-Hsiung, Chia-Yi, 621, Taiwan, ROC

[c] Institute of Oceanography, National Taiwan University, Taipei, 106, Taiwan, ROC

ARTICLE INFO

Article history:
Received 16 January 2008
Received in revised form 9 April 2008
Available online 9 May 2008

Keywords:
Long-range connection
Long-range interaction
Sandpile model
Precusory phenomena
Intermittent criticality
Seismicity
b-value

ABSTRACT

Reduction in *b*-values before a large earthquake is a very popular topic for discussion. This study proposes an alternative sandpile model being able to demonstrate reduction in scaling exponents before large events through adaptable long-range connections. The distant connection between two separated cells was introduced in the sandpile model. We found that our modified long-range connective sandpile (LRCS) system repeatedly approaches and retreats from a critical state. When a large avalanche occurs in the LRCS model, accumulated energy dramatically dissipates and the system simultaneously retreats from criticality. The system quickly approaches the critical state accompanied by the increase in the slopes of the power-law frequency-size distributions of events. Afterwards, and most interestingly, the power-law slope declines before the next large event. The precursory *b*-value reduction before large earthquakes observed from earthquake catalogues closely mimics the evolution in power-law slopes for the frequency-size distributions of events derived in the LRCS models. Our paper, thus, provides a new explanation for declined *b*-values before large earthquakes.

1. Introduction

Numerically simulating earthquakes is mainly based on conceptual models such as the spring-slider model of Burridge and Knopoff [7], the block structure model of Gabrielov et al. [12], the lattice-solid model of Mora and Place [26], and the sandpile model of Bak et al. [1]. The spring-slider model [7] and the sandpile model [1] are two types of simple cellular automata models among them. Bak et al. [1] originally introduced the self-organized criticality (SOC) concept. The study of the SOC has been mainly based on simulations using sandpile model. Bak et al. showed their sandpile model can reach a critical state without the need to fine-tune the system parameters. Their model has large avalanches randomly in critical state. The sandpile model can drive itself into a statistically stationary state characterized by spatial and temporal correlation functions exhibiting power-law behavior. The model has no characteristic avalanche size. It covers all possible values with power-law distribution. The exponent of the power law between avalanches size and frequency could be characteristic of the self-organized criticality ([1,2,9,27]).

Seismic activity is caused by stress accumulation in a rock. The rock fractured by stress accumulation releases energy to the surrounding materials. The surrounding material in the *critical* state that cannot bear the energy increase would set off a chain of events, namely a large earthquake, caused by energy release from neighboring sites. The universal Gutenberg–Richter power-law relation describes the energy distribution of earthquakes. Gutenberg and Richter in 1956

* Corresponding author. Tel.: +886 3 422 7151 65653; fax: +886 3 422 2044.
E-mail address: chencc@ncu.edu.tw (C.-c. Chen).

0378-4371/$ – see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2008.05.004

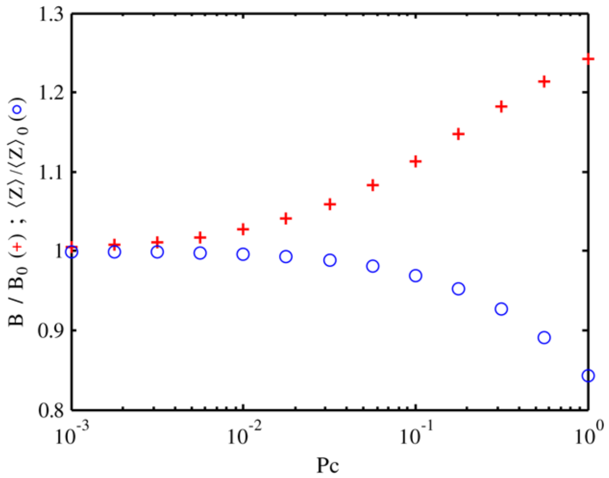

Fig. 1. Scaling exponents (crosses) of the long-range connective sandpile models with various connective probabilities P_c gradually increasing from 0 to 1 [10]. All the scaling exponents are normalized to the value of scaling exponent, i.e. $B_0 = 1.06$, of the nearest neighbor sandpile with $P_c = 0$. Also shown in the plot is the average number of total grains staying on the grid (circles) for each P_c. Again, the total grain number is normalized to the number for the case of $P_c = 0$, which is averagely $\langle Z\rangle_0 = 5244$.

noted earthquake frequency where energy larger than $\sim 10^{1.5M+16.1}$ dyne-cm is released, following a simple power law, $\log N_{\geq M} = a - bM$. The scaling exponent in the power-law Gutenberg–Richter relation is very well known as the *b*-value. The literature often discusses variation in the *b*-value and considers it a monitoring index related to forthcoming large earthquakes ([31,34,36,38,15,14,19,40,18,16,4,28,41,20,39]). Laboratory experiment results modeling natural faults also clearly show a lower scaling exponent during the foreshock period than during background periods ([37]). Several researchers have reported *b*-value reductions before large earthquakes ([33,21,23,32]).

Our present paper presents an alternative model reminiscent of real seismic activity, based on the original sandpile model ([1]). The modified sandpile model long-term averagely also has characteristics of power-law frequency-size distribution. The current work proposes an alternative sandpile model variant with random internal connections to demonstrate *intermittent criticality* and reductions in the scaling exponents before large avalanches.

2. Long-range connective sandpile models

We currently build a sandpile model by a very simple set of rules similar to the original Bak–Tang–Wiesenfeld-type sandpile model [1]. For a square grid of L by L cells, we randomly throw sands, one at a time, onto the grid. In the original sandpile model, once the total amount of accumulated sands within a single cell reaches the threshold amount of 4, they are redistributed to four adjacent cells (the nearest neighbors) or lost off the edge of the grid. All cells receiving grains from their neighbors are checked, and redistribution continued further away if any of them reaches over the threshold. Redistribution for each throw of new grains proceeds until none of the meshes receiving new grains exceeds the threshold. The total amount of cells involved in the redistribution process initiated by a single throw is defined as the size of the event. Note that the total amount of grains retained within the grid increases linearly in the beginning, transient thousands of iterations, and then stays at a quasi-static value with small fluctuations.

The modified rule of random internal connections is implemented by slightly changing the redistribution procedure as the following. When accumulated grains for any particular cell exceed the threshold and redistribution occurs, one of the original nearest neighbor connections faces a *connective probability* P_c, $0 < P_c < 1$, of redirecting to a randomly chosen, distant cell, replacing the original connection by a randomly chosen mesh that might be faraway from the toppling cell. We may call such a sandpile model version the *long-range connective sandpile* (LRCS) model. Apparently, when $P_c = 0$, the LRCS model reduces to the original nearest neighbor sandpile model of Bak et al. [1]. Findings show that the connective probability P_c has significant impact on scaling exponents of power-law frequency-size distributions in the LRCS model ([10]). Chen et al. [10] perform a series of simulations each with 10^6 throws of single grain on a square grid with 50 by 50 meshes, starting with the original nearest neighbor sandpile ($P_c = 0$) and then gradually increasing the connective probability P_c. Each P_c, robustly emerging has the frequency-size power law and the manifested SOC state. Their investigation finds a systematic steady increase of the scaling exponent along with a notable decrease in total grain amount staying on the grid (Fig. 1) when connective probability P_c gradually increases from 0 to 1.

In recent years, there have been numerous attempts to combine sandpile models with complex networks. Some papers have also studied the role of non-locality in sandpile behavior [25,17]. In addition, there have been attempts to apply sandpile with time-varying exponents and non-trivial connectivity to seismology and earthquake prediction [35]. In this study, more different from previous studies is essentially that we give an additional rule of self-adaptedly tuning P_c. We