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Surface charge carrier motion on insulating surfaces: One dimensional motion

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Abstract

We consider a model for the motion of charge carriers on the surface of an insulator. The insulator surface is either infinite, semi–infinite against a conducting half space or a strip between two conducting half spaces. The charge flux on the surface is assumed equal to the charge density times the electric field component in the surface, with time a constant. When the charge carrier motion in the plane is assumed constant in one direction, we can write the problem as an inviscid Burgers equation for a complex function. The imaginary part of this function is minus the carrier density while the real part, the Hilbert transform of the carrier density, is minus the electric field on the surface. Using the method of characteristics, we find an exact implicit solution for the problem and illustrate it with several examples. One set of examples, on the real line, or half of it, show how charge moves and how the surface may discharge into a conducting wall. They also show that the system can sustain shock wave solutions which are different from those in a real Burgers equation and other singular behaviour. Exact solutions on a finite strip between two conducting walls also show how that system can discharge completely, and also demonstrate shock waves. These systems are of particular interest because they are experimentally accessible.

Keywords: Charge motion on surfaces; One dimensional; Hilbert transform; Labert W function

1. Introduction

Previous attempts [1–3] to model charge carrier motion on the surface of insulators via the Mott–Gurney mechanism [4] have given reasonably good agreement with experimental data. This mechanism of charge motion on an insulator (dielectric) surface assumes that the current **J** of a mobile charged species on the dielectric surface is entirely driven by a particle current of the form

$$\mathbf{J}(x, y, t) = \mu N(x, y, t)\mathbf{E}(x, y, t) \tag{1.1}$$

where N is the time and space dependent particle density of charge carriers and **E** is the component of the electric field in the plane of the dielectric surface. The representation of the current **J** in Eq. (1.1) is just Ohm's law, with a resistivity expressed in terms of the carrier density.

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In reducing the motion to one dimensional motion, it was assumed (in Refs. [1–3]), that the electric potential satisfies a one dimensional Poisson equation. The resulting equations for the charge carrier density are non-linear partial differential equations which may be solved by the method of characteristics, that is by finding the curves in the (x, t) plane on which the carrier density (or electric field) is constant.

Similar modelling of circularly symmetric carrier distributions on circular samples of dielectrics (with an earthed edge) has led to similar "reasonable" agreement with experiment [5].

One of the critical assumptions in this earlier work on one dimensional motion was that the electric potential in the plane dielectric surface satisfies a one dimensional Poisson equation. In this paper we rederive the equations based on proper electrostatics and find similar equations for the potential, field and carrier density, but for a rather different object which involves Hilbert transforms of these quantities. The purpose of the paper is to develop solutions for a variety of circumstances in which the equations may be solved exactly.

In Section 2, we define the model more precisely and derive the basic equations for the system. In Section 3, we develop general properties of the solutions. In Section 4, we derive solutions on the whole real line, or half of it (with a conducting wall) when the initial charge carrier density is Lorentzian or the derivative of a Lorentzian, a rectangular pulse or a periodic function, while in Section 5 we consider exact solutions when the charge carrier density is on a strip between two conducting walls and demonstrate the way such strips can discharge completely, and in so doing, how they can sustain shock waves. The paper concludes with a discussion in Section 6.

2. The model and the basic equations

First we assume that the charge carrier density is entirely confined to the surface plane of the insulator or dielectric. In a precise model, of course, charged particles will not be precisely confined to this plane. One would expect that charged carrier particles can be slightly above the surface (in a region of dielectric constant 1 or (perhaps less, probably) slightly below the dielectric surface. This distribution of charge carriers normal to the plane may nonetheless be expected to be constrained to the plane on an extremely small length scale. Thus the models we develop will in fact be asymptotic expansions in a parameter increasing with this length scale. Thus the equations we develop for our model of charge carrier motion are leading order asymptotic expansions of a full description.

The electric field obeys a three dimensional Poisson equation

$$\nabla \cdot \mathbf{E} = \frac{4\pi Q}{\varepsilon(z)} N(x, y, t)\delta(z) \tag{2.1}$$

where we have assumed that the charge carrier particles carry a charge Q, N is their two dimensional density on the surface and $\delta(z)$ is the usual Dirac delta function. The dielectric constant $\varepsilon(z) = 1$ for $z > 0$ while $\varepsilon(z) = \varepsilon$ for $z < 0$, and we expect ε to be a large number for an insulator. The electric field $\mathbf{E}$ is related to a three dimensional potential Φ by the three dimensional equation

$$\nabla \Phi(\mathbf{r}, t) = -\mathbf{E}(\mathbf{r}, t) . \tag{2.2}$$

The carrier density and particle current $\mathbf{J}$ satisfy a two dimensional equation of continuity

$$\frac{\partial}{\partial t} N(x, y, t) + \frac{\partial}{\partial x} J_x(x, y, t) + \frac{\partial}{\partial y} J_y(x, y, t) = 0 \tag{2.3}$$

and the current is related to the charge carrier density and the plane component of the electric field by Eq. (1.1) which takes the form

$$\left(J_x(x, y, t), J_y(x, y, t)\right) = \mu N(x, y, t) \left(E_x(x, y, t), E_y(x, y, t)\right) . \tag{2.4}$$

Next, we assume that the carrier density, field and potential are independent of y, so that $E_y = 0$.

Using the Green's function for charges confined to the surface of an insulator, we then find that the x component of the electric field is

$$E_x(x, t) = \frac{2Q}{\varepsilon + 1} \int_{-\infty}^{\infty} \mathrm{d}x' N\left(x', t\right) \int_{-\infty}^{\infty} \mathrm{d}y \frac{x - x'}{\left((x - x')^2 + y^2\right)^{3/2}} = -\frac{4\pi Q}{\varepsilon + 1} P \frac{1}{\pi} \int_{-\infty}^{\infty} \mathrm{d}x' \frac{N\left(x', t\right)}{x' - x} \tag{2.5}$$