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Statistical analysis of the evolutionary minority game with different capacities

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Abstract

In this paper we consider the evolutionary minority game with different capacities. With the increase of capacity level ρ_1, the probability γ_n of room1 with n agents changes from one normal distribution to two normal distributions and back to one normal distribution again. When room2 is the basic strategy, the probability of room1 with n agents is a small normal distribution. And when room1 is the basic strategy the probability of room1 with n agents is a large normal distribution. We calculate the integral of the normal distributions and explain the characters in the figures. For instance, the room with more capacity represents the interest of the majority and the room has more probability to win.

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1. Introduction

Complex adaptive systems are ubiquitous in social, biological and economic sciences. Challet and Zhang [1,2] proposed a minority game (MG) to study the generic behavior of these complex systems. MG and its evolutionary version have been investigated in various papers in recent years [3–11]. The evolutionary minority game (EMG) consists of an odd number of agents. The agents choose whether to be in room1 or in room2 at each time step. At the end of each round, the agents belonging to the minority group win, each gaining R points (reward), while the agents belonging to the majority group lose 1 points (fine). When the reward-to-fine ratio $R/1$ is 1 (i.e. $R = 1$) the agents self-segregate into two opposing extreme group. When the reward-to-fine ratio $R/1$ is small enough the agents congregate at the middle cautious state [4]. This shows that the complex behavior can arise from relatively simple mathematical rules.

In EMG the two rooms are the same for the agents. While in many competition systems the choice objects are always different. For example different businesses have different profits and traders choose a high profit investment;

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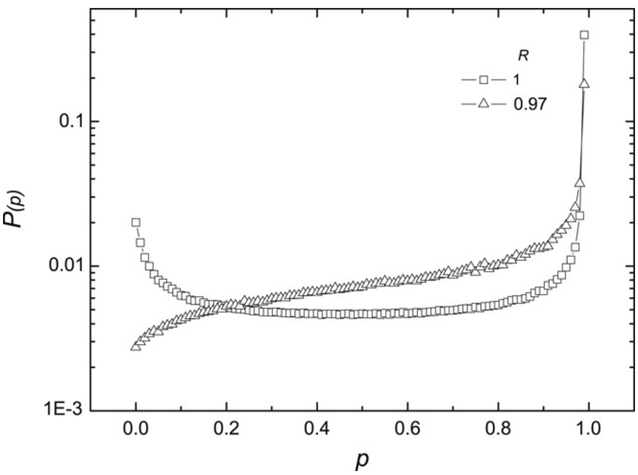

Fig. 1. Population distribution $P(p)$ with $N_1 = 700$, $N_2 = 300$, $\Delta p = 1$, $R = 1$ and $R = 0.97$.

different roads need different times and drivers choose a time-saving road. The economist, Pigou, brought forward the following case in 1924. There are two routes (I and II) from city A to city B. Route I takes 5 minutes while route II takes 30 min. We can imagine that the number of drivers who take route I will increase until the two routes need the same time, at which point the two routes have different capacities. Suppose it needs t min if n_1 drivers take route I and it needs the same t min if n_2 drivers take route II. Now there are $n_1 + n_2 + 1$ drivers. What will they do? In EMG, the agents choosing room is just like the drivers choosing route or the traders choosing investment. In this paper we introduce the different capacities to EMG and study the behavior of the agents (or drivers).

2. Model

We introduce the different capacities to EMG [4–7]. N agents repeatedly choose to be in room1 or in room2. The capacity of room1 is N_1 and that of room2 is N_2. We define$N_1 + N_2 + 1 = N$. The capacity level of room1 is $\frac{N_1}{N} = \rho_1$, while $\frac{N_2}{N} = \rho_2$ is the capacity level of room2. Agents in the room with the number of agents no more than its capacity win and get R point ($R \leq 1$). Agents in the room with the number of agents more than its capacity lose 1 point. All the agents share the common memory containing the winning room number. Suppose the memory length $m = 4$. Then, the sequence of the winning room number in the history is

$$\ldots\ldots 2112112122212122112211\underline{\underline{2121}}\underset{\cdot}{1}2112\underline{2121}$$

For recent winning room number 2121 (underlined), the basic strategy is the winning room number 1 (with under dot) next to the nearest same history 2121 (with double underlines). At each time step, each agent chooses the basic strategy in the memory with probability p, or the opposite alternative with probability $1 - p$. p is called the agent's gene value. If an agent's score falls below some value d ($d < 0$), its gene value is modified. The agent's new gene value is chosen randomly in the range $[\max(p - \Delta p, 0), \min(p + \Delta p, 1)]$, where Δp is called the adjustable range of the gene value. Then its score is reset to zero and the game continues.

3. Population distribution $P(p)$

The population distribution $P(p)$ is defined as the distribution of the agents in the gene value. When the two rooms have equal capacities ($N_1 = N_2$), the agents have no direct interaction but compete to be in the minority. In this case, when $R = 1$ the population distribution $P(p)$ self-segregates into two opposing groups ($p = 0$ or $p = 1$). With the decrease of R, the population distribution $P(p)$ changes to congregate at cautious state $p = 0.5$. In other words, the population distribution $P(p)$ is symmetrical (see Refs. [3–6]). When the two rooms have different capacities ($N_1 \neq N_2$), the agents should have direct interaction. Suppose room1 has larger capacity than room2 ($N_1 > N_2$). Then, what will happen to the population distribution? Fig. 1 shows the population distribution $P(p)$ at $N_1 = 700$ and $N_2 = 300$. It can be seen that when the two rooms have different capacities the population distribution is asymmetrical.