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Synchronization transition in gap-junction-coupled leech neurons[☆]

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Abstract

Real neurons can exhibit various types of firings including tonic spiking, bursting as well as silent state, which are frequently observed in neuronal electrophysiological experiments. More interestingly, it is found that neurons can demonstrate the co-existing mode of stable tonic spiking and bursting, which depends on initial conditions. In this paper, synchronization in gap-junction-coupled neurons with co-existing attractors of spiking and bursting firings is investigated as the coupling strength gets increased. Synchronization transitions can be identified by means of the bifurcation diagram and the correlation coefficient. It is illustrated that the coupled neurons can exhibit different types of synchronization transitions between spiking and bursting when the coupling strength increases. In the course of synchronization transitions, an intermittent synchronization can be observed. These results may be instructive to understand synchronization transitions in neuronal systems.

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Keywords: Coupled neurons; Synchronization transition; Oscillation; Leech neuron model; Bifurcation diagram

1. Introduction

Synchronization of a set of interacting individuals or units has been intensively studied because of its ubiquity in the natural world [1]. The synchronization of neuronal signals has been proposed as one of the mechanisms to transmit and code information in the human brain [2,3]. Mammalian nervous systems exhibit a diversity of synchronized behaviors including periodic, quasi-periodic, chaotic, noise-induced and noise-enhanced synchronous rhythms [4–8]. It was suggested that theoretical studies of such synchronized behaviors in neuronal assemblies play an

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important role in our understanding of information processing in the nervous systems. Hence, the synchronous firing of interconnected neurons has been extensively investigated by means of the theory of nonlinear dynamical systems. In Ref. [9] chaos synchronization in gap-junction-coupled neurons was studied by means of transversal and tangential Lyapunov exponents. It was shown that in strong gap junctions, the synchronous state was stable over a wide range of parameters irrespective of whether the synchronous state was chaotic or periodic. Effects of the width of an action potential on synchronization phenomena were investigated by the integrate-and-fire neuron model and the piecewise-linear version of the FitzHugh–Nagumo neuron model. It was shown that the duration of the impulse acted as a critical role in assuring synchronization [10]. By introducing weak heterogeneity to small-world networks of spiking neurons and using a semianalytical dynamical mean-field approximation, the synchronization may be slightly increased for diffusive couplings, while it is decreased for sigmoid couplings [11]. Synchronization of fast-spiking (FS) neurons interconnected by GABA-ergic and electrical synapses was investigated by Nomura and his co-workers [12]. It was observed that an FS neuron pair connected by electrical and chemical synapses could achieve both synchronous and antisynchronous firing states in a physiologically plausible range of the conductance ratio between electrical and chemical synapses. Noise-induced and noise-enhanced synchronizations including internal and external noise, have also been studied for neuronal systems [13,14]. Moreover, spatially collective motions in excitable neuronal media have been extensively studied and some novel results were shown in Ref. [15]. Synchronization in elliptic bursters was investigated and some analytic results was obtained in Ref. [16].

It is well known that neuronal oscillations can modulate cortical excitability and are critically involved in almost every cognitive task including information coding, memory formation and perception [6,17–19]. Neuronal oscillations result from the activity of individual neurons that can be roughly classified into two categories, being tonic spiking and bursting [20]. In addition, many neurons can display transitions between tonic spiking and bursting as a function of the brain state (e.g., sleep versus wakefulness); these transitions can modify the transfer mode of a neuronal population. Also, spiking and bursting modes of neuronal oscillations may co-exist under different initial conditions [21,22]. The study of synchronization on neuronal networks can also been seen in many recent references [[23,24] etc.] and the references therein.

To the best of our knowledge, there is very few reference available on synchronization dynamics of coupled multi-stable neuronal systems, thus it remains an interesting and challenging problem to be solved in the field of neuronal dynamics. In particular, dynamical mechanisms of underlying synchronization transitions of tonic spiking and bursting oscillations, as well as conditions that trigger transitions between these patterns of neuronal activity for the coupled neurons with co-existing attractors, is crucial for understanding processing of sensory information in the brain.

In the present paper, we are concerned with the synchronization process of coupled neurons that are characterized by co-existing spiking and bursting attractors as the coupling strength increases. It will be shown that the coupled neurons with co-existing attractors can exhibit different synchronization transitions between tonic spiking and bursting, which depend on the initial states of the uncoupled neurons. Even more interestingly, we will show that there exist narrow regions of the coupling strength, where an intermittent synchronization can occur when the coupled neurons transit from non-synchronization to synchronization or from spiking to bursting synchronization. Although our results are obtained for two coupled neurons, they can be generalized to multi-unit systems as well, thus shed light on the universal features of synchronization transitions in the neuronal apparatus.

The rest of this paper is organized as follows: model descriptions are introduced in Section 2. Main results of synchronization transitions in two coupled leech neurons with co-existing attractors are presented in Section 3, while a brief conclusion is given in Section 4.

2. Model descriptions

It is unknown what happens when one bistable neuron is coupled with another identical bistable neuron. The answer to this question manifests inherent difficulties. In what follows, it is demonstrated that dynamics of coupled bistable neurons are actually much richer and complicated, including different types of transitions between spiking and bursting synchronization, as well as intermittency.

To illustrate the above final statement, one may employ a minimal dynamical system, consisting of two leech neurons [22] that are coupled via a gap-junctional flux, and study their synchronization properties. The resultant dynamics of the coupled neurons are described by the following differential equations: