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Pseudoharmonic oscillator and their associated Gazeau–Klauder coherent states

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Abstract

In the paper we have constructed and examined the properties of the Gazeau–Klauder coherent states (GK-CSs) for the pseudoharmonic oscillator (PHO), one of three possible kinds in order to define the coherent states for this oscillator potential. In the second part, we have examined some nonclassical properties of these states. Our attention has been concentrated on the mixed states (thermal states). The diagonal P-representation of the corresponding density operator and some thermal expectations for the quantum canonical ideal gas of pseudoharmonic oscillators have also been examined. Like the CSs for the harmonic oscillator (HO), the GK-CSs for the PHO can be useful in the quantum information theory (QIT).

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1. Introduction

As it is well known that the real molecular vibrations are anharmonic, but due to its mathematical advantages, in many problems the harmonic oscillator (HO) model is used. Moreover, in other situations, it is compulsory necessary to use the anharmonic potential models. An anharmonic potential, which also permits an exact mathematical treatment is the so-called "pseudoharmonic oscillator" (PHO) potential, whose effective potential is [1–3]

$$V_j(r) = \frac{m\omega^2}{8} r_0^2 \left(\frac{r}{r_0} - \frac{r_0}{r}\right)^2 + \frac{\hbar^2}{2m} j(j+1) \frac{1}{r^2} \tag{1}$$

where m is the reduced mass, ω is the angular frequency of the PHO oscillator, while r_0 is the equilibrium distance between the diatomic molecule nuclei and $j = 0, 1, 2, \ldots$ is the rotational quantum number. This potential can be considered in a certain sense an intermediate potential between the harmonic oscillator potential (an ideal potential)

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and anharmonic potentials (the more realistic potentials). A comparative analysis of potentials HO-3D (3-dimensional harmonic oscillator potential) and PHO is performed in Ref. [3].

Using Molski's techniques [4] (for the Morse oscillator), in a previous paper [5] we have rewritten the PHO effective potential as follows:

$$V_j(r) = \frac{m\omega^2}{8} r_j^2 \left(\frac{r}{r_j} - \frac{r_j}{r}\right)^2 + \frac{m\omega^2}{4}(r_j^2 - r_0^2) \tag{2}$$

where the new appearing parameters are

$$r_j = \left[\frac{2\hbar}{m\omega}\left(\alpha^2 - \frac{1}{4}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}, \qquad \alpha = \left[\left(j + \frac{1}{2}\right)^2 + \left(\frac{m\omega}{2\hbar} r_0^2\right)^2\right]^{\frac{1}{2}}. \tag{3}$$

By using the substitution $\omega = 2\omega_0$ (the corresponding one-dimensional harmonic oscillator HO-1D has the angular frequency ω_0) and passing to the dimensionless variable $y = (\frac{m\omega_0}{\hbar})^{\frac{1}{2}} r$, the corresponding rotational–vibrational Schrödinger equation for the reduced radial function $u_v^\alpha(r)$ ($v = 0, 1, 2, \ldots, \infty$ is the vibrational quantum number) is [5]

$$\left[-\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}y^2} + \frac{1}{2}y^2 + \frac{1}{2}\left(\alpha^2 - \frac{1}{4}\right)\frac{1}{y^2} - (2v + \alpha + 1)\right] u_v^\alpha(y) = 0. \tag{4}$$

So, the dimensionless Schrödinger equation for the reduced radial function $u_v^\alpha(r)$ may be written as

$$H_\alpha^{(\mathrm{red})}(y) u_v^\alpha(y) = e_v u_v^\alpha(y). \tag{5}$$

Here the dimensionless reduced Hamiltonian $H_\alpha^{(\mathrm{red})}$ of the PHO appears:

$$H_\alpha^{(\mathrm{red})}(y) \equiv -\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}y^2} + \frac{1}{2}y^2 + \frac{1}{2}\left(\alpha^2 - \frac{1}{4}\right)\frac{1}{y^2} \tag{6}$$

and $e_v = 2v + \alpha + 1$ are its dimensionless eigenvalues.

In the previous paper [5] we have demonstrated that the $SU(1,1)$ is the dynamical group associated with the bounded states of the PHO. The Lie algebra corresponding to the group $SU(1,1)$ is spanned by the three group generators $\{K_1, K_2, K_3\}$ and the Casimir operator C_2 for any irreducible representation is the identity times a number:

$$C_2 = K_3^2 - K_1^2 - K_2^2 = K_3^2 - \frac{1}{2}(K_+K_- + K_-K_+) = k(k-1)I \tag{7}$$

where $K_\pm = K_1 \pm \mathrm{i}K_2$ are the raising and lowering operators of the group $SU(1,1)$.

In this context, a representation of $SU(1,1)$ is determined by a single real number k (called the Bargmann index). An algebraic approach to the pseudoharmonic oscillator in two dimensions, as well as the exact resolution of the arbitrary D-dimensional Schrödinger equation for this oscillator were performed recently [6,7].

Here, we are interested only in the unitary irreducible representations known as positive discrete series $\mathcal{D}^+(k)$, where $k > 0$. The corresponding Hilbert space $\mathcal{H}_k$ is spanned by the complete orthonormal basis of the number states $|v, k\rangle$:

$$\langle v, k | v', k\rangle = \delta_{vv'}, \qquad \sum_{v=0}^{\infty} |v, k\rangle\langle v, k| = 1. \tag{8}$$

In Ref. [5] we have showed that the following connection between the rotational parameter α (3) and the Bargmann index k exists:

$$\alpha = 2k - 1, \tag{9}$$

i.e. the rotational parameter α plays the role of the Bargmann index. Later in this paper, we will use the k-index instead of the α-index.