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Projective market model approach to AHP decision making

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Abstract

In this paper, we describe market in the projective geometry language and give the definition of a matrix of market rate, which is related to the matrix rate of return and the matrix of judgements in the Analytic Hierarchy Process (AHP). We use these observations to extend the AHP model to the projective geometry formalism and generalise it to an intransitive case. We give financial interpretations of such a generalised model and propose its simplification. The unification of the AHP model and projective aspect of portfolio theory suggests a wide spectrum of new applications for such an extended model.

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0. Introduction

The formalism derived from algebraical rules of Venetian accountancy of projective real geometry [1] determines a natural setting for modeling the market phenomena in situations, when we can neglect the scale effects and the transaction costs. It allows us, thanks to the elegant construction of the Hilbert metric, to take into account symmetries specific for modeling phenomena and analyse interesting interpretations for mathematical properties of different types of non-Euclidean geometries. The Analytic Hierarchy Process proposed by Saaty [2] is one of the methods of multi-criterion decision making and offers the precise quantitative method of hierarchization of criterion valuation in situations of full comparability of variants. It involves decomposing a complex decision into a hierarchy of clusters and sub-clusters, comparing properties of each possible pair of elements in each cluster as a matrix and synthesizing of priorities. Therefore the AHP can be considered to be both a descriptive and a prescriptive model of decision making. It is, perhaps, the most widely used decision making method in the world today [3]. Its validity is based on thousands of actual applications in which the AHP results were accepted and used by the decision makers. We extend the AHP model to the projective geometry formalism of dual objects: price-lists and portfolios. A side effect of such a financial description of the model is the identification of deviation of opinion from the condition of transitivity [4,5]

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as the existence, close by articulated hierarchy of criterions, special "transaction costs" which express aversions and preferences assessing to the concrete comparisons [6]. We show that this feature of the model has direct connection to the matrix of capital flows used in modeling the capital changes with the help of the matrix rate [7]. It is worth to signal broad perspectives of application of such extended variant of the AHP model. This version has been successfully applied in prioritisation, resource allocation, public policy, health care, strategic planning and many more [3,8,9]. The unification of the AHP model and projective aspect of portfolio theory suggests that a whole spectrum of problems can be described in such extension of the AHP model.

In the first section, we describe some projective markets, we define the matrix of market rate and its connection to the matrix rate of return and the matrix of judgements in the AHP model. In the second, we generalise the AHP model to intransitive case and interpret deviations from the transitivity condition in the projective market language. Finally, we discuss some method of simplification of the AHP.

1. Description of the projective markets

A market, which we denote by G, can be a market of goods and/or obligations, criterions of judgement, information and so forth. G has a natural structure of N-dimensional linear space over the reals [1]. Elements of this linear space are called baskets. For any basket $p \in G$ we have a unique decomposition into the goods which make it

$$p = \sum_{\mu=1}^{N} p_\mu \mathrm{g}_\mu$$

in some fix basis of normalised unit goods $(\mathrm{g}_1, \ldots, \mathrm{g}_N)$. The element $\mathrm{g}_\mu \in G$ is the μth market good and the coefficient $p_\mu \in \mathbb{R}$ is called the μth coordinate of the basket and expresses its quantity. The market quotation U is a linear map $U(\mathrm{g}_\nu, \cdot) : G \to \mathbb{R}$ which assigns to every basket p its current value in units of g_ν:

$$(Up)_\nu = U(\mathrm{g}_\nu, p) = \sum_{\mu=1}^{N} U(\mathrm{g}_\nu, \mathrm{g}_\mu) p_\mu,$$

where $U(\mathrm{g}_\nu, \mathrm{g}_\mu)$ is the relative price a unit of μth asset given in units of νth asset. The elements $u_{\nu\mu} := U(\mathrm{g}_\nu, \mathrm{g}_\mu)$ for $\nu, \mu = 1, \ldots, N$ make a $N \times N$ matrix of market rate.

The most general form of the matrix $u_{\nu\mu}$ was considered in paper [7]. The matrix rate of return describes the evolution of multi-dimensional capital and are given as a sum of two matrices: a matrix of flows, because the sum of the elements of each column is equal zero, and a matrix of growths—diagonal matrix. This description does not depend on the choice of basic goods and therefore we can consider the complex extended space of baskets and the basis of complex eigenvectors of the matrix rates in which the evolution of every capital basket can be represented as a set of non-interacting complex capital investments. In this complex basis, we do not observe any flows of the capital, but autonomous growth of individual components of the basket only. Then a description of the capital evolution is the most easily. The baskets of the complex capital have the interpretation in the real basis due to the transition matrix. In the complex extended space of baskets the decomposition of the matrix rate into the matrix of flows and the matrix of growths can always be done, in such a way that the matrix of flows is zero.

In the AHP model $u_{\nu\mu}$ is a pairwise comparison matrix of judgements. AHP involves decomposing a complex decision into a hierarchy of goal, criterions, sub-criterions and alternatives comparing properties of each possible pair of elements in each level as a matrix and synthesizing of the priorities. Alternatives can be quantitative (goods) or qualitative (personal preferences). Data are collected from decision-makers in the pairwise comparison of alternatives can be based on a quantity-based judgement in kilos, metres, euro, or on a quality-based judgement—equal, strong, very strong and so on. In the second case, we have to convert these judgements into numbers, see Ref. [10]. The matrix of judgements is reciprocal $u_{\mu\nu} = (u_{\nu\mu})^{-1}$ and reflexive $u_{\nu\nu} = 1$. The major problem is to find weights which order the objects and reflect the recorded judgements. Saaty suggests [10] calculating the principal right eigenvector which corresponds to the maximum eigenvalue $\lambda_{\max}$ of the judgement matrix:

$$\sum_{\mu=1}^{N} u_{\nu\mu} w_\mu = \lambda_{\max} w_\nu, \qquad \sum_{\nu=1}^{N} w_\nu = 1.$$