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Transport of waves in disordered waveguides: A potential model

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Abstract

We study the statistical properties of wave transport in a disordered waveguide. We first derive the properties of a "building block" (BB) of length δL starting from a potential model consisting of thin potential slices. We then find a diffusion equation—in the space of transfer matrices that describe our system—which governs the evolution with the length L of the disordered waveguide of the transport properties of interest. The latter depend only on the mean free paths and on no other property of the slice distribution. The *universality* that arises demonstrates the existence of a *generalized central-limit theorem*. We have developed a numerical simulation in which the universal statistical properties of the BB found analytically are first implemented numerically, and then the various BBs are combined to construct the full waveguide. The reported results thus obtained are in good agreement with microscopic calculations, for both bulk and surface disorder.

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Keywords: Disordered waveguides; Quantum transport; Random processes

1. Introduction

The complexity of the wave interference phenomena that occur when a wave propagates through a disordered medium containing a random distribution of scatterers is of considerable interest in many fields of physics. The problem has seen a revived interest in relation to the phenomenon of localization [1].

Remarkable *statistical regularities* have been found in such systems, in the sense that the probability distribution for various macroscopic quantities involves a rather small number of relevant physical parameters only. In Ref. [2] it was shown that a limiting distribution of physical quantities indeed arises in the so-called *dense-weak-scattering limit* (DWSL) and within a particular class of models, the relevant physical parameter being the mean free path. This result constitutes a generalized *central-limit theorem* (CLT) and coincides with that of the maximum-entropy model of Ref. [3], which gives rise to a diffusion equation known as the DMPK

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equation (after Dorokhov [4] and Mello et al. [3]). The maximum-entropy approach can thus be interpreted as capturing the features arising from a CLT. For waveguides with bulk disorder the statistical description of the conductance given by DMPK is excellent, whereas for waveguides with surface disorder it is not satisfactory [5]. An alternative approach to the study of disordered conductors is the supersymmetry method [6]. This method is able, in principle, to deal with problems with surface disorder; however, not much has been done in this direction.

The motivation of this presentation is to review recent work in which the statistical properties of a ''building block'' (BB) of length δL are derived starting from a potential model and are then used to find the ''evolution'' with length of the expectation value of physical quantities. As we shall see, in the analysis to be presented the energy appears explicitly, in contrast to earlier publications. Our model is also suitable to study wave-transport problems in which the physics of the various modes is relevant, as is the case of waveguides with surface disorder, instead of bulk disorder. We shall also find a good description of the statistical properties of quantities that involve phases, which were not described at all in previous models. The reader is referred to Ref. [7] for a detailed discussion of the model presented here.

The paper is organized as follows. In the next section we find the statistical properties of a BB using, as a potential model, thin slices perpendicular to the direction of the waveguide, admitting an arbitrary variation of the potential in the transverse direction. In Section 3 these results are used to find a Fokker–Planck equation for the ''evolution'' with the waveguide length L of the expectation value of the physical quantities of interest. It turns out that the cumulants of the potential higher than the second are irrelevant in the end. This signals the existence of a generalized CLT: once the mean free paths are specified, the limiting diffusion equation is universal, i.e., independent of other details of the microscopic statistics. The results obtained for a BB have an intrinsic interest as well, since they can be used to describe the statistical scattering properties of thin slabs. We indicate in Section 4 a numerical procedure that was developed to simulate numerically the diffusion process in transfer-matrix space. We present some of the results that we have been able to obtain so far and compare them with microscopic solutions of the Schrödinger equation. The conclusions of this work are given in Section 5.

2. Statistical properties of the building block

We construct the BB as a sequence of $m \gg 1$ δ-potential ''slices'' which are assumed to be equidistant, their separation being d, while their strength obeys some statistical distribution, as will be explained below. The physical regime in which we shall work is such that the separation d between slices is much smaller than the wavelength λ of the incident wave, the thickness δL of the BB and the mean free path ℓ, i.e.,

$$d \ll \{\lambda, \delta L, \ell\}, \tag{1}$$

as shown schematically in Fig. 1. The rth δ-slice potential is defined as

$$U_r(x, y) = u_r(y)\delta(x - x_r). \tag{2}$$

Since the Schrödinger equation has to be solved with Dirichlet boundary conditions at the lateral boundaries, we introduce the ''transverse'' states

$$\chi_a(y) = \sqrt{\frac{2}{W}} \sin \frac{\pi a y}{W}, \tag{3}$$

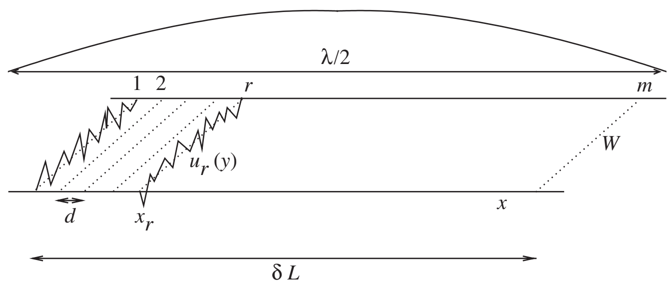

Fig. 1. Construction of the BB using δ-potential slices in the regime defined by inequality (1).