Physica A 390 (2011) 2229–2239

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

# The random field Ising model with an asymmetric trimodal probability distribution

I.A. Hadjiagapiou *

*Section of Solid State Physics, Department of Physics, University of Athens, Panepistimiopolis, GR 15784 Zografos, Athens, Greece*

ARTICLE INFO

*Article history:*
Received 14 July 2010
Received in revised form 13 December 2010
Available online 23 February 2011

*Keywords:*
Ising model
Mean-field approximation
Trimodal random field
Landau theory
Phase-diagram
Tricritical point
Phase transitions

ABSTRACT

The Ising model in the presence of a random field is investigated within the mean field approximation based on Landau expansion. The random field is drawn from the trimodal probability distribution $P(h_i) = p\delta(h_i - h_0) + q\delta(h_i + h_0) + r\delta(h_i)$, where the probabilities $p, q, r$ take on values within the interval $[0, 1]$ consistent with the constraint $p + q + r = 1$ (asymmetric distribution), $h_i$ is the random field variable and $h_0$ the respective strength. This probability distribution is an extension of the bimodal one allowing for the existence in the lattice of non magnetic particles or vacant sites. The current random field Ising system displays second order phase transitions, which, for some values of $p$, $q$ and $h_0$, are followed by first order phase transitions, thus confirming the existence of a tricritical point and in some cases two tricritical points. Also, reentrance can be seen for appropriate ranges of the aforementioned variables. Using the variational principle, we determine the equilibrium equation for magnetization, solve it for both transitions and at the tricritical point in order to determine the magnetization profile with respect to $h_0$.



## 1. Introduction

The pure models for crystalline materials can describe the respective experimental samples in exceptional situations, since such a sample can contain impurities, broken bonds, defects, etc., making real physical systems never translationally invariant, thus necessitating the modification of pure models appropriately for comparing the experimental results with the theoretical predictions. A small amount of quenched randomness can influence significantly the phase transitions replacing a first-order phase transition (FOPT) by a second-order phase transition (SOPT), so that tricritical points and critical-end-points are suppressed [1]. In two dimensions, an infinitesimal amount of field randomness destroys any FOPT. One such situation is the presence of random magnetic fields acting on each spin in an otherwise free of defects lattice; the respective pure system is considered to be described by an Ising model, which is now transformed into the random field Ising model (RFIM) in the presence of random fields [2–4]. RFIM had been the standard vehicle for studying the effects of quenched randomness on phase diagrams and critical properties of lattice spin-systems and had been studied for many years since the seminal work of Imry and Ma [4]. Associated with this model are the notions of lower critical dimension, tricritical points, higher order critical points and random-field probability distribution function (PDF). The simplest model exhibiting a tricritical phase diagram in the absence of randomness is the Blume–Capel model—a regular Ising spin-1 model [5,6]. Although much effort has been invested for the study of the RFIM, the only well-established conclusion is the existence of a phase transition for $d \geq 3$ ($d$ space dimension), that is, the critical lower dimension $d_l$ is 2 after a long controversial discussion [4,7], while many other issues are still unanswered; among them is the order of the phase transition, the existence of a tricritical point (TCP) and the dependence of these on the form of the random field PDF. According to the mean-field approximation (MFA) the choice

* Tel.: +30 2107276771; fax: +30 2107276711.
*E-mail address:* ihatziag@phys.uoa.gr.

0378-4371/$ – see front matter 
doi:10.1016/j.physa.2011.02.029

of the random field PDF can lead to a continuous ferromagnetic/paramagnetic (FM/PM) boundary as in the single Gaussian, whereas for the bimodal this boundary can be divided into two parts, an SOPT branch for high temperatures and an FOPT branch for low temperatures separated by a TCP at $kT_c^t/(zJ) = 2/3$ and $h_c^t/(zJ) = (kT_c^t/(zJ)) \times \arg\tanh(1/\sqrt{3}) \simeq 0.439$ [8–10], where $z$ is the coordination number, $k$ is the Boltzmann constant and $T_c^t, h_c^t$ are the tricritical temperature and random field, respectively, such that for $T < T_c^t$ and $h > h_c^t$ the transition to the FM phase is of first order. However, this behavior is not fully elucidated since in the case of the three dimensional RFIM, the high temperature series expansions yield only continuous transitions for both PDF's [11]; according to Houghton et al. [12] both distributions predict a tricritical point with $h_c^t = 0.28 \pm 0.01$ and $T_c^t = 0.49 \pm 0.03$ for the bimodal and $\sigma_c^t = 0.36 \pm 0.01$ and $T_c^t = 0.36 \pm 0.04$ for the Gaussian with critical standard deviation $\sigma_c^t$. Galam and Birman studied the crucial issue for the existence of a TCP within the mean-field theory for a general PDF $p(\overrightarrow{H})$ by using an even-degree free energy expansion up to eighth degree in the order parameter; they proposed some inequalities between the derivatives of the PDF up to sixth order at zero magnetic field for the possible existence of a TCP [13]. In Monte Carlo studies for $d = 3$, Machta et al. [14], using the Gaussian distribution, could not reach a definite conclusion concerning the nature of the transition, since for some realizations of randomness the magnetization histogram was two-peaked (implying an SOPT) whereas for other ones it was three-peaked implying an FOPT; Middleton and Fisher [15], using a similar distribution for $T = 0$, suggested an SOPT with a small order-parameter exponent $\beta = 0.017(5)$; Fytas et al. [16], following Wang–Landau and Lee entropic sampling schemes for the bimodal distribution function with $h_0 = 2$ and $h_0 = 2.25$ for a simple cubic lattice, concluded that their results indicated an SOPT by applying the Lee-Kosterlitz free energy barrier method; Hernández and coworkers claim they have found a crossover between an SOPT and an FOPT at a finite temperature and magnetic field for the bimodal distribution function [17]. One of the main issues was the experimental realization of random fields. Fishman and Aharony [18] showed that the randomly quenched exchange interactions Ising antiferromagnet in a uniform field $H$ is equivalent to a ferromagnet in a random field with the strength of the random field linearly proportional to the induced magnetization. Also another interesting result found by Galam [19] via MFA was that the Ising antiferromagnets in a uniform field with either a general random site exchange or site dilution have the same multicritical space as the random-field Ising model with bimodal PDF.

The usual PDF for the random field is either the symmetric bimodal

$$P(h_i) = p\delta(h_i - h_0) + q\delta(h_i + h_0) \tag{1}$$

where $p$ is the fraction of lattice sites having a magnetic field $h_0$, while the rest fraction has a field $(-h_0)$ and $p = q = \frac{1}{2}$ [8,20,21], or the Gaussian, single or double symmetric,

$$P(h_i) = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left[-\frac{h_i^2}{2\sigma^2}\right]$$
$$P(h_i) = \frac{1}{2}\frac{1}{(2\pi\sigma^2)^{1/2}} \left\{\exp\left[-\frac{(h_i - h_0)^2}{2\sigma^2}\right] + \exp\left[-\frac{(h_i + h_0)^2}{2\sigma^2}\right]\right\} \tag{2}$$

with mean value zero and $(h_0, -h_0)$, respectively, and standard deviation $\sigma$ [9,22].

Galam and Aharony, in a series of investigations, presented a detailed analysis via mean field and renormalization group of a system consisting of $n$-component classical spins (finally choosing $n = 3$) on a $d$-dimensional lattice of a uniaxially anisotropic ferromagnet in a longitudinal random field extracted from a symmetric bimodal PDF ($p = q = 1/2$) without and with a uniform magnetic field along the easy axis, respectively [23,24]. The uniaxial anisotropy was chosen to be along the easy axis and the exchange couplings were of the form $J^{(2)} = aJ^{(1)}$, where $a$ is the anisotropy and $0 \le a \le 1$. Depending on the anisotropy (small, medium, large) a variety of phases (longitudinal, transverse, paramagnetic), critical, bicritical, critical-end points as well as a multicritical point (an intersection of bicritical, tricritical and critical-end-point lines) resulted. In addition to these purely theoretical investigations, Galam, proposed a model (diluted random field) in his attempt to reproduce some of the features in the phase diagram of the experimental sample consisting of the mixed cyanide crystals $X(CN)_xY_{1-x}$, where $X$ stands for an alkali metal (K,Na,Rb) and $Y$ a spherical halogen ion (Br,Cl,I); the dilution of the pure crystal $XCN$ is achieved by replacing $CN$ by the halogen ions $Y$ [25]. The pure alkali-cyanide $XCN$ crystal ferroelastic transition disappears at some concentration $x_c$ of the cyanide; its numerical value depends on both components $X, Y$. By choosing a model Hamiltonian (ferromagnetic Ising-type with nearest neighbor interaction) with dilution and a symmetric trimodal PDF for the random fields, Galam, using MFA, managed to predict the involved first and second order phase transitions with the interfering TCP as well as the respective concentration for a phase transition to occur depending on the procedure considered. The random fields were necessary because there were experimental evidences that below $x_c$ cyanide displayed orientational freezing and the random fields were used for fixing this orientation. The involved probability $p_t$ in PDF as well as the critical threshold $x_c$ were expressed in terms of microscopic quantities.

Recently, the asymmetric bimodal PDF (1) with $p \neq q$, in general, has also been studied in detail [26]. This study has revealed that for some values of $p$ and $h_0$ the PM/FM boundary is exclusively of second order; however, for some other ranges of these variables this boundary consists of two branches, a second order one and another of first order, thus confirming the existence of a tricritical point, whose temperature depends only on the probability $p$ in (1). In addition to these findings, the occurrence of reentrance has been corroborated as well as complex magnetization profiles with respect the random field strength $h_0$. For $p = q$, symmetric bimodal PDF, the results found by Aharony were confirmed [8].