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Stochasticity and non-locality of time

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Abstract

We present simple classical dynamical models to illustrate the idea of introducing a stochasticity with non-locality into the time variable. For stochasticity in time, these models include noise in the time variable but not in the "space" variable, which is opposite to the normal description of stochastic dynamics. Similarly with respect to non-locality, we discuss the delayed and predictive dynamics which involve two points separated on the time axis. With certain combinations of fluctuations and non-locality in time, we observe a "resonance" effect. This is an effect similar to stochastic resonance, which has been discussed within the normal context of stochastic dynamics, but with different mechanisms. We discuss how these models may be developed to fit a broader context of generalized dynamical systems where fluctuations and non-locality are present in both space and time.

Keywords: Delay; Noise; Prediction; Stochastic resonance; Time; Non-locality

1. Introduction

"Time" is a concept that has drawn a lot of attention from thinkers in virtually all disciplines [1]. In particular, our ordinary perception is that space and time are not the same, and this difference appears in various contemplations of nature. It appears to be the main reason why the theory of relativity, which has conceptually brought space and time closer to receiving equal treatment, continues to fascinate and attract thinkers from diverse fields. Moreover, issues such as the "direction" or the "arrow" of time [2] and complex time [3] are current research interests.

It seems that there are other manifestations of this difference. One is the treatment of noise or fluctuations in dynamical systems. Time in dynamical systems, whether they are classical, quantum, or relativistic, is commonly viewed as not having stochastic characteristics. In stochastic dynamical theories, we associate noise and fluctuations with only "space" variables, such as the position of a particle, and not with the time variable. In quantum mechanics, the concept of time fluctuation is embodied in the time–energy uncertainty principle. However, time is not treated as a dynamical quantum observable, and clear understanding of the time–energy uncertainty has yet to be found [4].

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Another difference seems to show up in our cognition of non-locality in space and time. Non-local effects in space are incorporated in physical theories describing wave propagation, fields, and so on. In quantum mechanics, the issue of spatial non-locality is more intricate, constituting the backbone of such quantum effects as the Einstein–Podolsky–Rosen paradox [5]. With respect to time, there have been investigations of memory effects in dynamical equations. However, less attention has been paid to non-locality in time, and behaviors associated with non-locality in time, such as delay differential equations, are not yet fully understood [6–9].

Against this background, the main topic of this paper is to consider stochasticity and non-locality of time in classical dynamics through a presentation of simple models. We discuss delayed and predictive dynamics as examples of non-locality in time. For stochasticity, we present a delayed dynamical model with fluctuating time, or stochastic time. We shall see that this combination of stochasticity and non-locality in time can exhibit behaviors which are similar to stochastic resonance [10–12], which arises through a combination of oscillating behavior and "spatial" noise and has been studied in variety of fields [13–17].

2. Delayed and predictive dynamics

We start with a consideration of non-locality of time in classical dynamical models. The general differential equation of the class of dynamics we discuss here is as follows:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = f(\bar{x}(\bar{t}), x(t)). \tag{1}$$

Here, x is a dynamical variable of time t, and f is the "dynamical function" governing the dynamics. Its difference from the normal dynamical equation appears in $\bar{t}$, which can be either in the past or the future, and $t \neq \bar{t}$ in general. In other words, the change in $x(t)$ is governed by f, not its "current" state $x(t)$, but its state $\bar{x}$ at $\bar{t}$. We can define $\bar{t}$ and $\bar{x}$, as well as the function f, in a variety of ways. In the following, we will present two cases: delayed and predictive dynamics.

Delayed dynamics can be obtained from the general definition by

$$\bar{t} = t - \tau, \quad \bar{x}(\bar{t}) = x(t - \tau). \tag{2}$$

Here, $\tau > 0$ is the delay, and the dynamics depend on two points on the time axis separated by τ. Delayed dynamical equations have been studied for various applications [6–9].

Predictive dynamics, on the other hand, have recently been proposed [18,19], and they take $\bar{t}$ in the future, i.e., $\bar{t} = t + \eta$. We call $\eta > 0$ an "advance". We also need to define the state of the dynamical variable x at this future point in time. Here, we estimate x such that

$$\bar{x}(\bar{t} = t + \eta) = \eta \frac{\mathrm{d}x(t)}{\mathrm{d}t} + x(t). \tag{3}$$

This prediction is termed "fixed rate prediction". Namely, we estimate x as the value that would be obtained if the current rate of change extends for a duration from the present point to the future point. Qualitatively, this is one of the most commonly used methods for estimating population, national debt, and so on.

We also note that there are studies of equations called "functional differential equations of the advanced type", or "advanced functional differential equations" [20–22]. They also are differential equations with advanced arguments, and we can obtain equations of this class from our general definition by setting

$$\bar{x}(\bar{t} = t + \eta) = x(t + \eta), \tag{4}$$

with suitably chosen boundary conditions. The predictive dynamical equations differ from this class of equations, as we allow flexibility in defining $\bar{x}$ based on a prediction scheme.

We shall investigate the properties of these delayed and predictive dynamical models through computer simulations. To avoid ambiguity and for simplicity, we will study time-discretized map dynamical models, which incorporate the above-mentioned general properties of the delayed and predictive dynamical equations:

$$x_{n+1} = (1 - \alpha)x_n + f[\bar{x}_{\bar{n}}]. \tag{5}$$