Available online at www.sciencedirect.com
ScienceDirect
Physica A 379 (2007) 681–690
PHYSICA A
www.elsevier.com/locate/physa

Consensus problem in multi-agent systems with physical position neighbourhood evolving network ☆

Zheng Ping Wu*, Zhi-Hong Guan, Xianyong Wu

Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan 430074, PR China

Received 26 June 2006; received in revised form 14 November 2006
Available online 22 December 2006

Abstract

In multi-agent system (MAS), the communication topology of agent network plays a very important role in its consensus problem. To describe the communication topologies of MAS, a class of evolving network models with the concept of physical position neighbourhood connectivity are proposed and studied in this paper. The analysis and simulation results for network parameters such as the first nonzero eigenvalue and maximal eigenvalue of graph Laplacian matrix, clustering coefficients, average distances and degree distributions for different evolving parameters of these models are presented. The dynamical behaviour of each node on the consensus problem is also studied. It was found that the time to reach consensus becomes shorter sharply with the increasing of neighbourhood depth of the nodes in these models. And it was also found that for the maximal distance preferential attachment model (Model 3), the synthetic characteristic, such as robustness to communication delay, as well as convergence speed in consensus problem, is the best in all these models.

Keywords: Multi-agent systems; Complex network; Communication; Consensus problem; Physical position neighbourhood evolving mechanism

1. Introduction

In recent years, there have been significant researches in the area of coordinated control of MAS (Multi-Agent System). Examples include spacecraft formation flying [1,2], unmanned air vehicles (UAV) formation flying [3], formation control for underwater vehicles [4], automated highway systems control, and coordinated path planning [5]. As for the coordinated control of MAS, each agent's capability of information sharing is critical to the improvement of global performance. Information sharing entails the convergence of all local information vectors to a common value and we say that agents have reached agreement or consensus. This shared variable or information is a necessary condition for cooperation in MAS and can be interpreted differently depending on the application. It can present the attitude alignment in multiple spacecraft setting, heading direction in flocking, group average in distributed computation, etc.

☆Supported by the National Natural Science Foundation of China under Grants 60573005, 60603006 and 60628301.
*Corresponding author.
E-mail address: hnwzp@sina.com (Z.P. Wu).

0378-4371/$ - see front matter
doi:10.1016/j.physa.2006.12.026

By using graph theory and other methods, a number of researchers have addressed the multi-agents consensus problem under different communication topologies. The work in Ref. [6] focuses on attitude alignment on undirected graphs. It is shown that the consensus on the heading angles of the agents can be achieved if the union of the interaction graphs for the team are connected frequently enough as the system evolves. Average-consensus problem is solved in Ref. [7] over strongly connected and balanced graphs. Ren et al. [8] extended the results of Ref. [6] from the bidirectional case to unidirectional case. If the union of the collection of interaction graphs across some time intervals has a spanning tree frequently enough, consensus (not necessarily average-consensus) can still be achieved. In Ref. [9], tools from nonlinear dynamics are used to obtain a broad class of communication patterns that guarantee global consensus. In Ref. [10], by studying the time the agents reach consensus in small-world network communication topologies, Reza Olfati Saberst points out that a consensus problem can be solved hundreds of times faster on certain small-world communication topology than on nearest-neighbour coupled one.

In most of the existing work on multiple-agents cooperative control, the availability of a communication link between any two agents is always assumed. However, this assumption may not be true in many situations. The communication range of each agent is limited. The links between agents are always determined by the physical positions of the agents and each agent only communicates with its physical position neighbourhood agents, i.e., the agents which are within its communication range. The communication bandwidth of agent is also limited, thus limiting the amount of information that can be exchanged. The communication links between the agent pairs may be broken as a result of obstacle, enemy or other reason. How do we build or rebuild the communication network of MAS? Motivated by these questions, a class of physical position neighbourhood evolving network models are established to describe the communication topologies of MAS in this paper. It is assumed that the agent network is formed by evolving. At the beginning, there only exist a small number of m_0 agents, and e_0 communication links between the agents. Then the new agents join in the agent network one by one. For any new agent i, any other current agent within communication range of the agent i will be considered as its physical position neighbourhood agent. For simplicity, the number of physical position neighbourhood agents of agent i is fixed as M in this paper, and the new agent i only communicates with these M agents. It is also assumed that when a new agent joins in the agent network, it will create m links with m different agents within its communication range. When $M>m$, the new communication links can be selected from these M agents by the following five different methods: random, minimal physical distance preferential, maximal physical distance preferential, maximal node degree preferential and minimal node degree preferential. Based on the above assumptions, a class of physical position neighbourhood evolving network models are established for describing the communication topologies of MAS. In these models, the communication connected agents are regarded as a coupled complex network, where each node represents an agent, and each edge between two nodes represents a communication link between two agents. The individual agent shares a common state space and each agent updates its current state based on the information received from other agents according to a simple rule. Our objectives are to relate the information flow and communication structures (the models of complex network) with communication delay and convergence speed of group agents consensus, and to study how much effect a class of physical position neighbourhood models have on their degree distribution, eigenvalue distribution, clustering coefficients, average distance and so on.

The remaining of this paper is organized as follows. In Section 2, two cases of the models of MAS' consensus problem, with and without time-delay, are both presented. A lemma is also presented in this section. The new physical position neighbourhood evolving network models are introduced in Section 3. Based on the models constructed in Section 3, the degree distribution, eigenvalue distribution and the consensus problem behaviour of the new models are studied in Sections 4, 5 and 6, respectively. Finally, conclusions are made in Section 7.

2. Consensus problem in complex network

Similar to Ref. [10], consider a network of integrator nodes $\dot{x}_i = u_i$ with topology $G=(V,E)$ in which each node only links with its neighbouring nodes $N_i=\{j \in V : \{i,j\} \in E\}$ on $G=(V,E)$. Here, $V=\{1,2,\ldots,n\}$ and $E \subset [V]^2$ (the set of 2-element subsets of V) denote the set of nodes and edges/links of the network,