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Clustering and maximal flow in vehicular traffic through a sequence of traffic lights

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Abstract

We study the maximal current (maximum traffic capacity) of vehicular traffic through a sequence of traffic lights on a highway, where all signals turn on and off synchronously. The dynamical model of vehicular traffic controlled by signals is expressed in terms of a nonlinear map, where the excluded-volume effect is taken into account. The dynamical behaviors of vehicles are clarified by analyzing traffic patterns. The clustering of vehicles varies with the cycle time of signals. The maximum current is closely connected to vehicular clustering. Clustering of vehicles is controlled by varying both split and cycle time of signals. The dependence of the maximal current on both split and cycle time is derived.

Keywords: Traffic dynamics; Clustering; Many-particle system; Signal control; Nonlinear map; Complex system

1. Introduction

Traffic flow is a kind of many-particle system of strongly interacting vehicles. Recently, transportation problems have attracted much attention in the fields of physics [1–5]. The traffic flow and its related problems have been studied from a point of view of statistical mechanics and nonlinear dynamics [6–25]. Interesting dynamical behaviors have been found in the transportation system.

Mobility is nowadays one of the most significant ingredients of a modern society. In urban traffic, vehicles are controlled by traffic lights to give priority for a road because the city traffic networks often exceed the capacity. Brockfeld et al. [26] have studied optimizing traffic lights for city traffic by using a CA traffic model. They have clarified the effect of signal control strategy on vehicular traffic. Also, they have shown that the city traffic controlled by traffic lights can be reduced to a simpler problem of a single-lane highway. Sasaki and Nagatani have investigated the traffic flow controlled by traffic lights on a single-lane roadway by using the optimal-velocity model [27]. They have derived the relationship between the road capacity and jamming transition.

Traffic depends not only on traffic quantity (or density) but also on both cycle time and split of traffic signals. Here the cycle time is the period of a traffic light and the split of signal is the fraction of green time to

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the signal period. Until now, one has studied the periodic traffic controlled by a few traffic lights. It has been concluded that the periodic traffic does not depend on the number of traffic lights [26,27]. Very recently, a few works have been done for the traffic of vehicles moving through an infinite series of traffic lights. The effect of both cycle time and split on vehicular traffic has been classified [28–31]. When the traffic quantity increases, the traffic current increases at low density while it saturates at high density. The saturated current is the maximal one that vehicles are possible to move through a series of traffic signals. The maximal current depends on the traffic pattern. The pattern changes by varying both split and cycle time of signals. The clustering of vehicles occurs by controlling the signals. The clustering is connected to the maximal current. However, the dependence of maximal current on the signal characteristics is little known.

In this paper, we study the maximal current of vehicular traffic moving through an infinite series of traffic lights. The signals are periodically positioned with a constant distance on a single-lane roadway, controlled by the synchronized strategy, and turn on or off with a cycle time and a split. We present an extended model of nonlinear map to take into account the excluded-volume effect of vehicles. We show that the clustering of vehicles occurs by controlling the signals. We clarify the connection between the vehicular clustering and the maximal flow.

2. Nonlinear-map model

We consider the flow of vehicles going through an infinite series of traffic lights. Each vehicle is inhibited to pass other vehicles. Size of each vehicle is $l_{\min}$. A vehicle moves with the mean speed v if the way is not blocked by other vehicles and there are no signals. Then, the minimal time headway is $l_{\min}/v$. We consider the vehicular traffic on one-dimensional lattice. We set the lattice spacing as the minimal headway. The site is occupied by a single vehicle or empty. The overlapping of vehicles at a site is inhibited. Thus, the excluded-volume effect is taken into account. The traffic lights are periodically positioned with distance M. The lattice sites are numbered, from upstream to downstream, by 1, 2, 3, …, n, $n+1$, …. Also, vehicles are numbered, from the leader to the rear, by 1, 2, 3, …, i, $i+1$, …,N.

In the synchronized strategy, all the traffic lights change simultaneously from red (green) to green (red) with a fixed time period $(1-S_p)T_s$ (S_pT_s). The period of green is S_pT_s and the period of red is $(1-S_p)T_s$. Time T_s is called the cycle time and fraction S_p represents the split which indicates the ratio of green time to cycle time. When a vehicle arrives at a traffic light and if the traffic light is red, the vehicle stops at the position of the traffic light. Then, when the traffic light changes from red to green, the vehicle goes ahead. On the other hand, when a vehicle arrives at a traffic light and if the traffic light is green, the vehicle does not stop and goes ahead without changing speed.

We define the arrival time of vehicle i at the site $nM+1$ just after traffic light M as $T(i,nM+1)$. The arrival time of vehicle i is given by

$$\begin{aligned} T(i,nM+1) = {} & T(i,nM) + 1 + (R(i,nM) - T(i,nM)) \\ & \times H(T(i,nM) - \left(\text{int}(T(i,nM)/T_s)T_s\right) - S_pT_s), \\ \text{with} \quad & R(i,nM) = (\text{int}(T(i,nM)/T_s) + 1)T_s, \end{aligned} \tag{1}$$

$$T(i,nM+1) = \max[T(i,nM+1), T(i-1,nM+2)], \tag{2}$$

where $H(T)$ is the Heaviside function: $H(T)=1$ for $T \geqslant 0$ and $H(T)=0$ for $T<0$. $H(T)=1$ if the traffic light is red, while $H(T)=0$ if the traffic light is green. $R(i,nM)$ is such time that the traffic light just changed from red to green. The third term on the right hand side of Eq. (1) represents such time that the vehicle stops if traffic light nM is red.

If vehicle i reaches the vehicle ahead $i-1$, it does not pass over the proceeding and it keeps the minimal headway. Eq. (2) represents the condition of no passing.

In the region except for the signals, the arrival time of vehicle i at the site $nM+m$ ($1<m\leqslant M$) is given by

$$T(i,nM+m) = T(i,nM+m-1) + 1, \tag{3}$$

$$T(i,nM+m) = \max[T(i,nM+m), T(i-1,nM+m+1)]. \tag{4}$$