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Schematic models for fragmentation of brittle solids in one and two dimensions

F.P.M. dos Santos, R. Donangelo, S.R. Souza*

Instituto de Física, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, 21941-972 Rio de Janeiro, RJ, Brazil

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Abstract

Stochastic models for the development of cracks in 1D and 2D objects are presented. In one dimension, we focus on particular scenarios for interacting fragments (IF1) and non-interacting fragments (NIF1) during the breakup process. For 2D objects, we consider only NIF1, but analyze isotropic and anisotropic development of fissures. Analytical results are given for many observables. Power-law size distributions are predicted for some of the fragmentation pictures considered.

Keywords: Fracture; Size distributions; Power law

1. Introduction

The breakup of a system into many pieces, i.e., fragmentation, is a subject of intensive investigation in different areas of science and engineering [1–4]. The interest in this phenomenon ranges from studies of the observed size distributions of atomic nuclei [2,3] and chains of molecules [4] to asteroids [5], ice floes [6–8], brittle solids [9–14], thin glass plates [15], eggshells [16,17], frozen potatoes [18], fluids [13], drops in general [19,20], etc. The size distributions observed in the breakup of these systems exhibit, in general, a power-law behavior, suggesting, in some cases, at least, that the process is related to critical phenomena.

This peculiarity has led to the development of many schematic models which, to a large extent, disregard microscopic details of the fragmenting macroscopic objects. The fact that the size distributions seem to be fairly insensitive to the constituent elements of the fragmenting body [18] gives strong support to this approach. Therefore, different treatments, based on quite general assumptions, have been proposed to explain the characteristics of the fragmentation of brittle solids. These approximations include, for instance, mean-field treatments [21–24], fractal analysis [15,25], dynamical models of granular solids [26], random forces stopping models [27], quantum tunneling [28], and schematic branching–merging models [29,30] (for a recent review see Ref. [1] and references therein).

In this work, we present schematic models for the breakup of rods and flat brittle objects. For the latter case, our models are intended to describe the fragmentation of objects which suffer a strong impact on one

*Corresponding author.
E-mail address: srsouza@if.ufrg.br (S.R. Souza).

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of its sides, in contrast to those proposed in Refs. [21–24,29], in which the stress is uniformly distributed inside the body. Effects associated with anisotropy are also investigated. In the 1D case, we compare the properties of the fragments produced when the interaction between them can be neglected, i.e., when they are ejected from the parent body, with the case in which they continue interacting during the breakup process.

In Section 2 we present the models and make analytical predictions related to the size distributions, whereas in Section 3 we discuss and interpret the results. Conclusions are drawn in Section 4.

2. The models

The main characteristic of the models presented below is that they can be viewed as stochastic processes, in which strict mass conservation is imposed in each event. We present below their detailed formulation.

2.1. 1D models

Objects whose lengths are large compared with their cross-sections are represented by 1D segments of a line. Thus, in this subsection we consider fractures on a line of unitary length. The number of fractures N is related to the violence of the impact on the object. More specifically, N points $\{x_i\},\ i = 1, \ldots, N$, are associated with the fractures and two adjacent points delimit a fragment, so that $N + 1$ fragments are formed at the end of the process.

We concentrate on two different fragmentation models, which aim at describing distinct breakup scenarios:

(i) Non-interacting fragments (NIF1), where fracture points are sequentially generated and the $(i+1)$th crack can only appear at the right-hand side of the ith fissure. It is numerically implemented by selecting the first random point x_1 uniformly in the interval (0,1). Then, the next crack point x_2 is uniformly chosen in the interval $(x_1,1)$, and so on. This corresponds to a simplified picture for the breakup of an object when the impact zone is concentrated close to its left edge, similar to the (almost) perpendicular fall of a rod. In this case, the dynamics cannot produce further fractures in fragments that have already been released from the parent fragment.

(ii) Interacting fragments (IF1), in which the broken pieces exert stress on the others, leading to further cracks. For simplicity, at the ith step, one of the $i + 1$ fragments is chosen with equal probability and a fissure point is sampled uniformly inside the selected fragment.

The analytical construction of the NIF1 model can be achieved through the following considerations:

(1) When the ith crack point is made, there exists an equivalence between the fragment just formed and the remaining of the parent body.

(2) Let us assume that, after the ith fracture, the parent body has length l. Then, as the $(i+1)$th fissure is produced, the probability of it having length χ must be inversely proportional to its parent's length l.

Upon denoting by $P_i(\chi)$ the probability density of creating a fragment of size $0<\chi<1$ at the ith crack, one may then write

$$P_{i+1}(\chi) = \int_{\chi}^{1} \frac{1}{l} P_i(l)\,\mathrm{d}l. \tag{1}$$

By noticing that $P_1(\chi) = 1$, this recurrence relation can be iterated, leading to

$$P_i(\chi) = \frac{[-\log(\chi)]^{i-1}}{(i-1)!}, \tag{2}$$