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The oscillation of stock price by majority orienting traders with investment position

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Abstract

We consider an interacting particle system for the stock price fluctuation. The change of the stock price with a feedback by the price considering the herding behavior (majority orienting behavior) of traders, gives the van der Pol equation as a deterministic approximation. Considering the investment position of each trader, we introduce the delayed van der Pol equation. The history of investment positions, for example sell or buy, of each trader for a stock makes a memory effect, which is modeled by using the time retardation. The delayed van der Pol equation model seems to be natural and explains typical phenomena, for example triangle pattern, volatility jumps, price jumps and price trends, known for the time series of a stock price.

Keywords: Stock price; Delayed van der Pol equation; Poincare–Bendixson theorem; Behavioral finance

1. Introduction

Using the Ising model, the analysis of the market has been discussed in Refs. [1,2]; besides, modeling the financial market by a certain form of Ising structure of the interactions of agents seems to be successfully achieved in several studies [3,4]. We think that most traders are influenced by rumors, excessively or under excessively react to the information, and like the subjective desirability more than the objective probability [5,6]. The minority traders of a market, who are diffident to their investment position in many cases, are going to follow the decision of the majority. Because they tend to think that the majority of the traders have more accurate information than themselves. A majority orienting model [7] is introduced, which is composed of three elements: the mutation of dealers, the majority rule and the feedback by the price, as basic elements for the change of a stock price in a real market. This model is a ternary interaction model of a finite particle, which makes excursions that are similar to the Ising model [8], assuming a mutation to the other type for each particle. The van der Pol equation is obtained as a deterministic approximation, which seems to explain the oscillation of a stock price.

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Traders make use of information from the history of a stock price in order to gain profits by dealing stocks. We develop the majority orienting model taking into account of the feedback rule considering the history of buying then selling of traders and introduce a delayed van der Pol equation. Our present model makes the majority orienting model more realistic and helps to understand the dynamics for the change of the stock price. Our model seems to give an explanation for the typical phenomena, known for the chart analysis of the time series of a stock price, which is commonly used by traders; for example, triangular pattern volatility jumps, price trends and price jumps, those are perturbed by random noise in a real market.

2. The majority orienting model

The van der Pol equation is obtained from the majority orienting model for the change of a stock price [7]. In the model there are two types of particles in a box plus (+) and minus (−), whose numbers are N_+ and N_-, respectively, with $N = N_+ + N_-$. Let each trader be considered to be a particle in the box and change his position at random by the following step, with three substeps (1), (2) and (3), which are successively applied to the particles in the box. Here a + particle represents a bullish (feeling confident about the future stock price) trader, while a − particle represents a bearish (feeling pessimistic about the future stock price) trader.

(1) *Mutation rule*: One particle out of N particles is chosen at random. It changes its sign to the opposite sign with probability m and does not change with probability $1-m$, $(0 \leqslant m \leqslant 1)$.

(2) *Majority rule*: Three particles are taken at random. If two of the particles taken have the sign + and one has the sign −, the one with − changes its sign to + and the price S increases by 1, while, if two of the particles have the sign − and one has +, the one with + changes to − and the price S decreases by 1. If the three particles have the sign +, no change of sign occurs for the three particles and the price S increases by 3, while, if the three particles have −, no change of sign occurs for the three particles and the price S decreases by 3.

(3) *Feedback rule*: If S is positive, N_+ is decreased by 1 with probability S/N, while, if S is negative, N_+ is increased by 1 with probability $-S/N$. The absolute value of S can be larger than N when m is small. We only discuss the case of $|S| \leqslant N$ in this section. This condition is almost valid when $m \geqslant 0.75$.

Let us represent N_+ and S at step s as $N_+(s)$ and $S(s)$, respectively. Assuming that the duration of a step is τ, and the values of $N_+(s)$, $N_-(s)$ and $S(s)$ are given, we have the following expected values:

$$E\left[\frac{N_+(s+1) - N_+(s)}{\tau N}\right] = m\left\{-\frac{N_+(s)}{N} + \frac{N_-(s)}{N}\right\} + 3\frac{N_+(s)(N_+(s)-1)N_-(s)}{N(N-1)(N-2)} - 3\frac{N_+(s)N_-(s)(N_-(s)-1)}{N(N-1)(N-2)} - v\frac{S(s)}{N} \quad (1)$$

$$E\left[\frac{S(s+1) - S(s)}{\tau N}\right] = 3\frac{N_+(s)(N_+(s)-1)(N_+(s)-2)}{N(N-1)(N-2)} + 3\frac{N_+(s)(N_+(s)-1)N_-(s)}{N(N-1)(N-2)} - 3\frac{N_-(s)(N_-(s)-1)N_+(s)}{N(N-1)(N-2)} - \frac{N_-(s)(N_-(s)-1)(N_-(s)-2)}{N(N-1)(N-2)}. \quad (2)$$

When N is sufficiently large, we obtain the deterministic approximation Eqs. (3) and (4), putting $(N_+(s) - N/2)/N$ as x_t and $S(s)/N$ as y_t, taking an appropriate time scale τ as

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}x_t &= -2mx_t + 6x_t\left(\frac{1}{2} + x_t\right)\left(\frac{1}{2} - x_t\right) - vy_t \\ &= -2\left(m - \frac{3}{4}\right)x_t - 6x_t^3 - vy_t,\end{aligned} \quad (3)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}y_t = 6x_t. \quad (4)$$

Assuming the number of the particles N is large enough, we can neglect the random sampling effect of particles, while in the real market the stock price y_t is perturbed by random noise. Hence, we have the