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Quantum-phase transition in a XY model

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Abstract

In this paper we study quantum-phase transition in the one-dimensional XY model with an XY easy-plane single ion anisotropy. We use the path-integral formalism, but consider the effect of quantum fluctuations, which renormalize the parameters of the system, using the self-consistent harmonic approximation. We show that the quantum fluctuations increase the effective coupling constant of the model.

Keywords: Quantum-phase transition; XY model; One-dimensional

Quantum-phase transitions (QPT) have attracted much interest in the past decade, driven by experiments on the cuprate superconductors, the heavy fermions materials, organic conductors, and related compounds [1]. These transitions take place at the absolute zero of temperature, where crossing the phase boundary means that the quantum ground state of the system changes in some fundamental way. This is accomplished by changing not the temperature, but some parameter, let us say g, in the Hamiltonian of the system [2]. As pointed out by Sachdev [1] a traditional analysis of many-body system would begin from either a weak-coupling Hamiltonian, and then build in interactions among the nearly free excitations, or from a strong-coupling limit, where the local interactions can be taken into account in a precise way, but their propagation is not fully understood. On the other side, a quantum critical point starts from an intermediate coupling regime and provides another perspective on the physics of the system. There are two possibilities for the $T>0$ phase diagram of a system near a quantum critical point. In the first the singularity in the free energy is present only at $T=0$ at a quantum critical point at $g=g_C$. This will be the case for the one-dimensional model studied here. In the second, there is a line of $T>0$ phase transition that terminates at g_C. The physics of the quantum transition is in general complex. One theoretical model where it can be well understood is the anisotropic quantum XY model, in one- and two-dimensions, described by the following Hamiltonian:

$$H = -J\sum_n (S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) + D\sum_n (S_n^z)^2, \tag{1}$$

where the parameter D represents an xy easy-plane single-ion anisotropy. This Hamiltonian model has some similarity with the one describing array of Josephson junctions connecting two superconducting metallic

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grains [3]. In contrast to the Heisenberg ferromagnet, the ground state of the XY model is no longer trivial. In fact, it shares many properties with the antiferromagnetic one-dimensional Heisenberg model.

In this paper, we will study the QPT in the Hamiltonian (1) in one-dimension. Low-dimensional systems are very convenient to study QPT since by changing the interactions in the system one changes the amount of quantum fluctuations. We consider the case $S=1$, and in this case the Berry phase can be ignored [4]. Our calculations differ mainly from former calculations by the inclusion of quantum fluctuations, which renormalize the parameters of the system.

The spectrum of the Hamiltonian (1) is expected to change drastically as D varies from very small to very large values. Papanicolaou [5] has studied the case of strong planar anisotropy, the so-called large D phase, using strong-coupling methods. This phase consists of a unique ground state with total magnetization $S^z_{\text{total}} = 0$ separated by a gap from the first excited states, which lie in the sectors $S^z_{\text{total}} = \pm 1$. The primary excitation is a gapped $S=1$ exciton with an infinite lifetime at low energies. For small D, the Hamiltonian (1) is in a gapless phase well described by the spin-wave formalism. The Hamiltonian is invariant under the transformation $J \to -J$, and a shift of the Brillouin zone $k \to k+\pi$.

The Hamiltonian (1) can be studied by means of a self-consistent harmonic approximation (SCHA). The SCHA replaces the Hamiltonian of a system by an effective Hamiltonian with temperature-dependent renormalized parameters [6]. The SCHA was used in the study of the one-dimensional quantum sine-Gordon problem, where it describes correctly the phase transition of the model. The reason is that it is equivalent to a renormalization group analysis to one loop [7]. Writing the spins components in the Hamiltonian (1) in terms of the Villain representation [8]:

$$
\begin{aligned}
S_n^+ &= \mathrm{e}^{\mathrm{i}\varphi_n}\sqrt{(S+\tfrac{1}{2})^2 - (S_n^z+\tfrac{1}{2})^2}\,,\\
S_n^- &= \sqrt{(S+\tfrac{1}{2})^2 - (S_n^z+\tfrac{1}{2})^2}\,\mathrm{e}^{-\mathrm{i}\varphi_n}\,,
\end{aligned} \tag{2}
$$

we obtain

$$
\begin{aligned}
H = &-J\tilde{S}^2 \sum_n \sqrt{1-(S_n^z/\tilde{S})^2}\sqrt{1-(S_{n+1}^z/\tilde{S})^2}\\
&\times \cos(\varphi_n - \varphi_{n+1}) + D\sum_n (S_n^z)^2\,,
\end{aligned} \tag{3}
$$

where $\tilde{S}^2 = S(S+1)$. Generalizing the procedure presented in Ref. [6] to the quantum case we find in the SCHA:

$$
H_0 = J\sum_n \left[\rho\frac{\tilde{S}^2}{2}(\varphi_n - \varphi_{n+1})^2 + (\rho+\delta)(S_n^z)^2\right], \tag{4}
$$

where $\delta = D/J$ and ρ is the stiffness constant, renormalized by quantum fluctuations, given by

$$
\rho = \left(1 - \left\langle (S_n^z/\tilde{S})^2 \right\rangle\right)\exp\left[-\tfrac{1}{2}\left\langle (\varphi_n - \varphi_{n+1})^2 \right\rangle\right]. \tag{5}
$$

To calculate ρ we will use the low D region where a spin wave treatment can be used. Taking the Fourier transform of Eq. (4) we find:

$$
H_0 = J\sum_q \left[\rho\frac{\tilde{S}^2}{2}(1-\cos q)\,\varphi_q\varphi_{-q} + (\rho+\delta)S_q^z S_{-q}^z\right]. \tag{6}
$$

We remark that Eq. (4) contains more information about the physics of the problem than Eq. (6). In Eq. (4) φ is considered as an angle variable and therefore topological excitations can be present, while in Eq. (6) φ is just a standard field. In Eq. (4) we have phenomena, which are not accessible to perturbation theory. It becomes essential then to include macroscopic quantum fluctuations, in the form of topological excitations, where the fields perform large excursion from the ground state. The most important role of topological