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Is the expression $H = 1/(3 - q)$ valid for real financial data?

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Abstract

Non-extensive thermodynamics is one of the most intriguing physics new frontiers. A large number of researchers have been successfully finding connections between the new concepts introduced by this new field and other complex systems already presented. In particular, Borland [Phys. Rev. E 57 (1998) 6634–6642] has introduced a very interesting relation between the entropic index q that arises in the non-extensive entropy and the well-known Hurst exponent H used to measure long-range dependence in complex systems. In this paper, we provide statistical support to Borland results and test the validity of these results in real financial data.
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1. Introduction

Non-extensive thermodynamics is one of the most intriguing physics new frontiers. Besides tackling to situations not considered by the usual thermodynamics, its tools have provided a new way to face dynamic complex systems [1–3]. Moreover, a large number of researchers have been successfully finding connections between the new concepts introduced by this new field and other complex systems already presented. One of the areas greatly beneficiated by these new tools is Econophysics. Several papers have dealt with economics and financial issues in the context of non-extensive thermodynamics. For example, one may find in a recent physics literature that stock returns distributions are very well fitted by the Tsallis distribution with an entropic index q around 1.5 [4,5], a measure of the risk of the stock returns using the Tsallis distribution [6], an option pricing model based on the Tsallis distribution [7] and some connections between the non-extensive thermodynamics and the behavioral economics [8,9].

Borland [10] has introduced a very interesting relation between the entropic index q that arises in the non-extensive entropy and the well-known Hurst exponent H used to measure long-range dependence in complex systems.

Long memory dependence is a very well-studied topic in the financial literature. One of the first to consider the existence of long memory behavior in asset returns was Mandelbrot [11]. Since then, many others have

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supported Mandelbrot's results. The presence of long-range dependence in asset returns is a serious drawback since it contradicts the weak form of market efficiency which states that, under the information contained on the set formed by past returns, future returns are unpredictable [12].

In this paper, on one hand, we extend the computational experiment found in Ref. [10] and provide statistical support to its results. On the other hand, we test the validity of these results in real financial data.

This paper is organized as follows. The Tsallis distribution and the role of the entropic index are introduced in Section 2. The methods used to evaluate the Hurst exponent are introduced in Section 5. The expression that relates the entropic index to the Hurst exponent is revised in Section 3. In Section 4, we present the measures of long-range dependence considered in this work. In Section 5, we expose the main results of this work. Finally, Section 6 presents some conclusions of this work.

2. The Tsallis distribution

There is no doubt that one of the most important contributions brought by non-extensive thermodynamics was the so-called non-extensive distributions.[1] The most common is the one that is found from the maximization of the non-extensive entropy [14]

$$S_q = k\left(\frac{1-\int p(x)^q\,\mathrm{d}x}{q-1}\right) \tag{1}$$

subjected to the usual constraints

$$\int p(x)\,\mathrm{d}x = 1, \tag{2}$$

$$\langle x-\overline{x}\rangle \equiv \int (x-\overline{x})p(x)^q\,\mathrm{d}x = 0, \tag{3}$$

$$\langle (x-\overline{x})^2\rangle \equiv \int (x-\overline{x})^2 p(x)^q\,\mathrm{d}x = \sigma_q^2, \tag{4}$$

i.e.,

$$p(x) = \frac{1}{z}(1+\beta(q-1)(x-\overline{x}))^{-1/(q-1)}, \tag{5}$$

where

$$z = \left(\frac{\pi}{\beta(q-1)}\right)^{1/2}\frac{\Gamma((3-q)/2(q-1))}{\Gamma(1/(q-1))} \tag{6}$$

is the appropriate normalization factor and $\Gamma(\cdot)$ is the gamma function.

If $q=1$, it is obvious that Eq. (5) recovers the gaussian distribution.

Furthermore, the ordinary variance is given by

$$\sigma^2 = \frac{1}{\beta(5-3q)} \tag{7}$$

for $q<\frac{5}{3}$ and it diverges for values $q\geqslant\frac{5}{3}$.

On the other hand, it is easy to show that the kurtosis coefficient K depends only on q and it is given by

$$K = \frac{3(5-3q)}{7-5q}. \tag{8}$$

Eq. (8) is particularly useful to calculate q from real data.

[1] See [13] for a comprehensive presentation of these distributions.

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