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Physica A 373 (2007) 785–795
PHYSICA A
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# Avalanches in complex spin networks

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Received 30 January 2006; received in revised form 26 April 2006
Available online 21 June 2006

**Abstract**

We investigate the magnetization reversal processes on classes of complex spin networks with antiferromagnetic interaction along the network links. With slow field ramping the hysteresis loop and avalanches of spin flips occur due to topological inhomogeneity of the network, even without any disorder of the magnetic interaction [B. Tadić, et al., Phys. Rev. Lett. 94 (2005) 137204]. Here we study in detail properties of the magnetization avalanches, hysteresis curves and density of domain walls and show how they can be related to the structural inhomogeneity of the network. The probability distribution of the avalanche size, $N_s(s)$, displays the power-law behavior for small $s$, i.e., $N_s(s) \propto s^{-\alpha}$. For the scale-free networks, grown with preferential attachment, $\alpha$ increases with the connectivity parameter $M$ from 1.38 for $M = 1$ (trees) to 1.52 for $M = 25$. For the exponential networks, $\alpha$ is close to 1.0 in the whole range of $M$.


*Keywords:* Antiferromagnets; Barkhausen noise; Evolving networks; Ising model; Monte Carlo simulations; Simple graphs and trees; Spin dynamics

## 1. Introduction

Growing networks is a vivid area of interdisciplinary sciences, with long list of applications [1–6]. The mainstream of literature is concentrated on the structure of the networks. However, from the point of view of some applications it is of interest to decorate nodes with additional degrees of freedom. In the simplest case, these variables are discrete, with two possible values: ±1. Such an extension can be useful when discussing numerous examples, from neural and logical networks through sexual networks to quantum gravitation [7]. Our topic here is a network of interacting magnetic moments (spins), with two possible magnetic orientations, up or down. The models of spin networks with ferromagnetic interaction have been considered in several recent studies [8,9]. The case of spin networks with antiferromagnetic interactions is essentially different because of the frustration effect along closed cycles on the graph [10,11]. This difference is well known also for regular lattices [12].

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0378-4371/$ - see front matter 
doi:10.1016/j.physa.2006.04.117

Recently, we introduced and investigated field-driven dynamics of spins on complex networks with the antiferromagnetic interaction [11]. We found that the structural complexity of the networks leads to avalanches of spin flips and a criticality of the hysteresis loop. The desired shape of hysteresis curves can be obtained by tuning the clustering parameter $M$. This parameter counts the number of links, which attach each new node to the growing network.

In this work we expand the study of the reversal processes in the antiferromagnetic spin networks. We demonstrate the genesis of avalanches due to inhomogeneity in network's connectivity when the clustering is low. In this limit the avalanche structure and duration can be clearly interpreted in terms of the theoretical distribution of connectivity and depth of the network. For large clustering, the character of the avalanche spectra $N_s(s)$ for large $s$ change from multipeaky to a soft decreasing. For small avalanches, the avalanche spectra display the power-law character in the whole range of the clustering parameter $M$. We define the size $s$ of avalanche as a number of spins which were flipped at a given value of external field $H$.

In Section 2 we introduce details of the model and discuss spin reversal on small graphs. Section 3 contains details of the emergent structures for two classes of growing networks exponential and scale-free. Our results on the magnetization reversal in these networks are given in Section 4 and discussed in Section 5.

## 2. Spin reversal on small graphs

To describe our procedure in details, we begin with small simple graphs (Fig. 1). A spin $S_i = \pm 1$ is placed at each node. If two nodes are linked together, antiferromagnetic interaction tends to keep them in one of two antiparallel configurations. Energy of the whole system is written as

$$E = -J \sum_{\langle ij \rangle} S_i S_j - H \sum_i S_i, \tag{1}$$

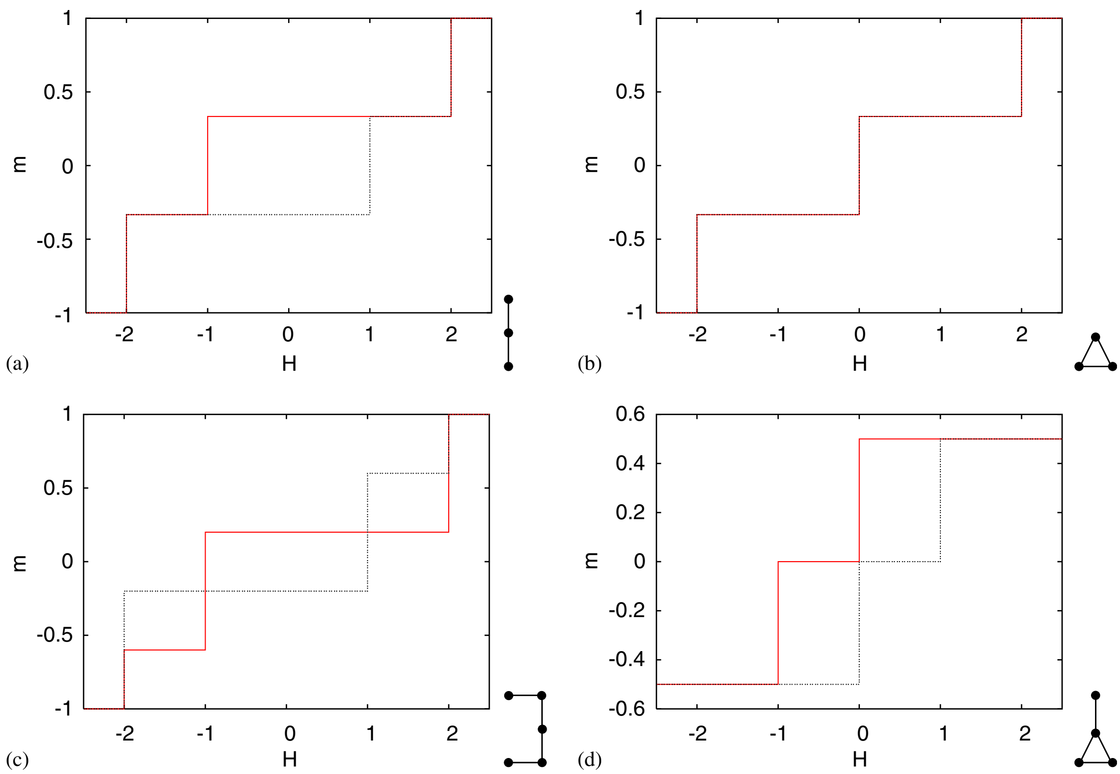


Fig. 1. Toy graphs and their hysteresis loops for decreasing field (continuous red line) and increasing field (dotted black line).