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Properties of transportation dynamics on scale-free networks

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Abstract

In this work, we study the statistical properties of transportation dynamics considering congestion effects, based on the standard Barabási–Albert scale-free model. In terms of user equilibrium (UE) condition, congestion effects can be described by cost function. Simulation results demonstrate that the cumulative load distribution exhibits a power-law behavior with $P_l \sim l^{-(\gamma-1)}$, where l is the flow loaded on the node and $\gamma \approx 2.7$ which is much bigger than that obtained in many networks without considering congestion effects. That is, there exist fewer heavily loaded nodes in the network when considering congestion effects. Furthermore, by numerically investigating overload phenomenon of the heaviest loaded link removal in transportation networks, a phase-transition phenomenon is uncovered in terms of the key parameter characterizing the node capacity.

Keywords: Scale-free networks; Transportation networks; Congestion effects; Overload phenomenon

1. Introduction

Scaling laws and patterns have been detected in a great number of systems found in nature, society, and technology, including the protein–protein interaction networks [1,2], food webs [3,4], scientific collaboration networks [5–7], sexual relations [8], transportation networks [9,10], and the World Wide Web [11–13], etc. Particularly, transportation phenomenon on complex networks is of vital importance in both theoretical and practical perspectives. Properties of transportation dynamics on scale-free networks considering congestion effects are investigated in this work.

Currently, the load was introduced to address the pattern of transport. In most studies, all links and nodes were viewed as identical in terms of their functional roles in the binary network [14–18]. Since the weights (e.g. cost) of links were various in realities, many researches considered the heterogeneity of elements [18–26]. Moreover, congestion effects were taken into account in Refs. [27,28], i.e., the weight of a link is a function of its cumulative load (called cost function) [29], which is common in transportation networks. Load distribution $P_L(l)$ considering congestion effects was studied on different complex networks, as detailed in Ref. [27], where

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l is the load on the node. Identical "practical capacity" for each link was considered in Ref. [27]. However, "practical capacities" of different links must be diversified, which is taken into account in this work, furthermore overload phenomenon is studied.

The rest of this paper is organized as follows: Section 2 provides a brief introduction about transportation dynamics. Simulations and results are given in Section 3. Finally, some summaries and conclusions are shown in Section 4.

2. Transportation dynamics

Networks comprise nodes and links, which represent the individuals and their interactions, respectively. We consider network with n nodes, and many vehicles simultaneously travel from different origins to different destinations (all nodes are both origins and destinations), in terms of minimum-cost routes. In order to compare with the results obtained in many networks, we assume that between each $r-s$ pair, the traffic volume $q_{rs}=1$. If all vehicles are to take the same path (which may initially be the optimal one in terms of cost), congestion will develop on it. As a result, the cost on this path may increase to a point where it is no longer the optimal path. Some of these vehicles will then use an alternative path. The alternative path can, however, also be congested, and so on. A stable condition is reached only when there is no incentive for route switching. This is the characterization of user equilibrium (UE) condition, where the load pattern can result from joint decisions by all vehicles to act so as to minimize a mathematical program [29]. The program can be expressed as follows:

$$\min_X \quad Z(X)=\sum_{(i,j)}\int_0^{x_{ij}} w_{ij}(x)\,\mathrm{d}x, \tag{1}$$

$$\text{s.t.} \quad \sum_k f_k^{rs}=q_{rs} \quad \forall r,s, \tag{2}$$

$$f_k^{rs}\geqslant 0 \quad \forall r,s,k, \tag{3}$$

$$x_{ij}=\sum_r\sum_s\sum_k f_k^{rs}\delta_{ij,k}^{rs} \quad \forall i,j, \tag{4}$$

where X is the matrix of link flow x_{ij} on the link (i,j), $w_{ij}(\cdot)$ is the cost function on the link (i,j), the path flow f_k^{rs} is the flow of the path k connecting origin r and destination s, $\delta_{ij,k}^{rs}=1$ if link (i,j) is a part of path k, and $\delta_{ij,k}^{rs}=0$ otherwise. Eq. (1) is the objective function, which is the sum of the integrals of the link performance functions viewed as a mathematical construct utilized to solve equilibrium problems. Eq. (2) represents a set of flow conservation constraints that the flow on all paths connecting each $r-s$ pair has to equal to its traffic volume. The nonnegativity conditions in Eq. (3) are required to ensure that the solution of the program will be physically meaningful. Eq. (4) means that the flow on each link is the sum of flows on all paths going through the link. Algorithm solving the mathematical program is given in Appendix A.

Congestion effects can be described by the cost function. Here we focus on the Bureau of Public Roads (BPR) formula [30], which is the most widely used in urban transportation networks. The BPR formula can be expressed as

$$w_{ij}=w_{ij}^0\left(1+\alpha\left(\frac{x_{ij}}{C_{ij}}\right)^{\beta}\right), \tag{5}$$

where $\alpha=0.15$ and $\beta=4$ are typically used. C_{ij} is the "practical capacity" of link (i,j). It is reasonable to assume that "practical capacity" of link (i,j) is around its link flow x_{ij} when UE situation reaches, which is taken into account in this work.[1] w_{ij}^0 denotes the cost on the link (i,j) when there are no flows. In order to compare with the results obtained in many networks, $w_{ij}^0=1$ for each link is used in our simulation.

[1]The "practical capacity" C_{ij} of link (i,j) is gradually iterated in our simulation. In detail, adjust the value of C_{ij}, $C_{ij}=x_{ij}$ if $\max_{(i,j)}|(x_{ij}/C_{ij})-1|>\xi$ $(\xi>0)$ when UE situation reaches, and then new flow to each link is loaded, and adjust again. The procedure is iterated until $\max_{(i,j)}|(x_{ij}/C_{ij})-1|\leqslant\xi$.