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Freezing transition in the mean-field approximation model of pedestrian counter flow

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ABSTRACT

We study the freezing transition in the counter flow of pedestrians within the channel numerically and analytically. We present the mean-field approximation (MFA) model for the pedestrian counter flow. The model is described in terms of a couple of nonlinear difference equations. The excluded-volume effect and bi-directionality are taken into account. The fundamental diagrams (current–density diagrams) are derived. When pedestrian density is higher than a critical value, the dynamical phase transition occurs from the free flow to the freezing (stopping) state. The critical density is derived by using the linear stability analysis. Also, the velocity and current (flow) at the steady state are derived analytically. The analytical result is consistent with that obtained by the numerical simulation.

1. Introduction

Recently, pedestrian and vehicular traffics have attracted considerable attention [1–5]. Many observed dynamical phenomena in pedestrian and traffic flows have been successfully reproduced with physical methods. The pedestrian flow dynamics is closely connected with the driven many-particle system [6]. It has also encouraged physicists to study the evacuation processes by driven many-particle models [7–13]. The pedestrian and vehicular traffic models have been applied to the traffic flow of such mechanical mobile objects as robots [14,15].

The typical pedestrian flows have been simulated by the use of a few models: the lattice–gas model of biased-random walkers [11–16], the molecular dynamic model of active walkers [6,10,17], and the cellular automaton model [7,8]. Helbing et al. have found that the "freezing by heating" occurs in the pedestrian counter flow by the use of the molecular dynamic model of active walkers [17]. By using the lattice–gas model of biased-random walkers, Muramatsu et al. have found independently that the jamming (freezing) transition occurs from the free flow to the freezing (stopping) state when the pedestrian density is higher than the critical value [16]. The jamming transition in the pedestrian counter flow has been studied by some researchers [18–21].

In the jamming transition, pedestrian flow in the crowd changes from the free traffic to the jammed traffic in which pedestrians are distributed heterogeneously and move slowly. In the freezing transition, pedestrian flow changes to the frozen state in which all pedestrians cannot move by preventing from going ahead each other. Thus, the freezing transition is definitely different from the jamming transition. The similar freezing transition occurs in the two-dimensional traffic cellular automaton model proposed by Biham, Middleton, and Levine [22,23].

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However, the theoretical analysis for the freezing transition has been little known. The pedestrian flow has been investigated only by the numerical simulation of the self-driven many-particle models. The mean-field approximation model is unknown for the pedestrian flow until now [1–3].

In this paper, we present the mean-field approximation (MFA) model for the pedestrian counter flow. We study the dynamical phase transition in the MFA model of pedestrian numerically and analytically. We derive the numerical and analytical solutions to the pedestrian counter flow. We present the fundamental diagram (current–density diagram) numerically and analytically. We apply the linear stability method to the MFA model. We derive the freezing transition point analytically. We compare the analytical result with the numerical result.

2. Mean-field approximation model

We consider the counter (bi-directional) flow of pedestrians in the channel. There exist two kinds of walkers within the channel: the one is the walkers moving to the east and the other the walkers moving to the west. The walker moving to the east (or west) interacts highly with the other walkers in the front. When the density of walkers ahead is higher, the current decreases more because the movement of walkers is prevented by other walkers.

We consider the mean-field approximation for the pedestrian counter flow. We approximate the counter flow on the square lattice as that on one-dimensional lattice because walkers to east or to west move uni-directionally on the average. We define the probability that a walker to east (to west) exists on site i at time t as $p_E(i,t)$ ($p_W(i,t)$). We apply the conservation law of probability $p_E(i,t)$ ($p_W(i,t)$) to the counter flow. The probabilities $p_E(i,t+\Delta t)$ and $p_W(i,t+\Delta t)$ of a walker to east and a walker to west existing on site i at time $t+\Delta t$ are described by the following:

$$p_E(i,t+\Delta t) = p_E(i,t) + \left[p_E(i-1,t)P_{t,E}(i-1\to i,t) - p_E(i,t)P_{t,E}(i\to i+1,t)\right]\Delta t, \tag{1}$$

$$p_W(i,t+\Delta t) = p_W(i,t) + \left[p_W(i+1,t)P_{t,W}(i+1\to i,t) - p_W(i,t)P_{t,W}(i\to i-1,t)\right]\Delta t, \tag{2}$$

where $P_{t,E}(i\to i+1,t)$ is the hopping probability of walker to east from site i to site $i+1$ at time t and $P_{t,W}(i\to i-1,t)$ is the hopping probability of walker to west from site i to site $i-1$ at time t. The second term on the right hand in Eq. (1) represents the inflow of a walker to east from site $i-1$ to site i between t and $t+\Delta t$. The third term represents the outflow of a walker to east from site i to site $i+1$ between t and $t+\Delta t$. Similarly, the second and third terms of Eq. (2) represents the inflow and outflow of a walker to west on site i between t and $t+\Delta t$.

The excluded-volume effect is represented by the probability that a site is occupied by other walkers. Hopping probability $P_{t,E}(i-1\to i,t)$ is proportional to the probability that site i is empty at time t. At a mean-field approximation, we approximate the hopping probabilities as follow:

$$\begin{aligned} P_{t,E}(i-1\to i,t) &= \left(1-(p_E(i,t)+p_W(i,t))^{\alpha}\right), \\ P_{t,W}(i\to i-1,t) &= \left(1-(p_E(i-1,t)+p_W(i-1,t))^{\alpha}\right). \end{aligned} \tag{3}$$

Here, exponent α describes the dependence of the hopping probability on the pedestrian density. It represents the strength of the excluded-volume effect. When exponent α is small (large), the dependence of hopping probability on density is high (small). Exponent α is smaller, the hopping probability is smaller with increasing density.

Eqs. (1)–(3) are a couple of nonlinear difference equations. It is not easy to obtain the analytical solution but possible to obtain the numerical solution. Also, it is able to derive analytically the velocity and current at a steady state. By applying the linear stability analysis to the counter flow, one can derive the transition point for the freezing analytically.

We derive the velocity and current at a steady state from Eqs. (1)–(3) analytically. If the freezing transition does not occur, Eqs. (1)–(3) reduce to the followings at the steady state

$$\begin{aligned} p_E(i-1)\left(1-(p_E(i)+p_W(i))^{\alpha}\right) &= p_E(i)\left(1-(p_E(i+1)+p_W(i+1))^{\alpha}\right), \\ p_W(i+1)\left(1-(p_E(i)+p_W(i))^{\alpha}\right) &= p_W(i)\left(1-(p_E(i-1)+p_W(i-1))^{\alpha}\right). \end{aligned} \tag{4}$$

In the free flow state at low density, the probabilities are given by the uniform solution. Therefore, the velocity and current are obtained

$$v_E = v_W = 1-(\rho_E+\rho_W)^{\alpha}, \quad j_E = \rho_E(1-(\rho_E+\rho_W)^{\alpha}), \quad \text{and} \quad j_W = \rho_W(1-(\rho_E+\rho_W)^{\alpha}), \tag{5}$$

where $v_{E(W)}$ is the velocity of walkers to east (to west) and $j_{E(W)}$ is the current of walkers to east (to west).

3. Simulation result

We carry out the numerical simulation for nonlinear difference equations (1)–(3). The boundaries are periodic. If Δt equals one, the difference equations do not produce the physical meaning solution but it presents the nontrivial solution when Δt is less than 0.1. When $\Delta t \le 0.1$, the numerical result depends little on the interval Δt. The steady state is independent on interval Δt. We set $\Delta t = 0.05$. The parallel update is used because our model is not CA but the difference equation.