Physica A 388 (2009) 4845–4855

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

The dynamics of exchange rate time series and the chaos game

C.P. Cristescu, C. Stan *, E.I. Scarlat

"Politehnica" University of Bucharest, 313 Spl. Independetei, Bucharest, RO- 060042, Romania

ARTICLE INFO

Article history:
Received 29 January 2009
Received in revised form 29 July 2009
Available online 13 August 2009

Keywords:
Exchange rate time series
Lempel–Ziv complexity
Chaos game
Iterated function systems
Gaussian noise

ABSTRACT

This work presents a novel method of reconstructing some relevant characteristics of exchange rate time series by the superposition of two components: a mostly deterministic one, the chaos game as expressed by the Yuan/USD exchange rate and a purely stochastic one, Gaussian white noise. We analyzed 20 economic systems with the average Index of Economic Freedom above 50. The considered characteristics (the Lempel–Ziv complexity index, the slimness of the distribution and the Iterated Function Systems clumpiness test) are well reproduced by the reconstruction process. Additional confirmation is obtained by an analysis of the exchange rate of the Romanian national currency as an example of an application of the method to a transition economy, and by an analysis of the time series of the Euro-zone as an example of an application to a multinational system using a shorter time series.

1. Introduction

The interest of scientists in the study of financial time series is over a century old. The explosive development of the theory of nonlinear phenomena stimulated the interest of physics scientists in the economic problems and the new interdisciplinary field of Econophysics emerged [1]. Two main streams can be identified in the analysis of financial time series presented in the literature. Some papers interpret the time evolution as exclusively random [2–4], while others try to explain the behavior in terms of nonlinear theory, implying chaotic dynamics [5–7].

In the present study we identify deterministic and stochastic contributions in the exchange rate time series and argue that the admission of the presence of both is essential for the understanding of some relevant aspects of the dynamics. The paper is organized as follows. Section 2 presents the data, their source and selection arguments. The next three sections introduce the methods used in the analysis, i.e., the box and whisker statistics, the Lempel–Ziv (L–Z) complexity and the Iterated Function Systems (IFS) clumpiness test, particularly the chaos game and the Sierpinski gasket. In Section 6 the deterministic and the stochastic contributions to the dynamics of exchange rate time series are identified and sustained. Section 7 presents the main restrictions on the Gaussian noise component imposed by the parameters that, in our analysis, characterize the exchange rate time series, i.e., the slimness and the L–Z complexity. The results of the analysis and their interpretation make the object of the following section. Finally, Section 9 consists of a synthetic review of the important results and points out the original contributions.

2. Data

The yearly Index of Economic Freedom (IEF) provides a picture of more than 180 countries against a list of 50 variables divided into 10 broad factors of economic freedom [8]. The equally weighted factors are aggregated in a single score, namely the IEF. According to the IEF analysis, the lower the score, the less freedom an economic system enjoys.

* Corresponding address: "Politehnica" University of Bucharest, Faculty of Applied Sciences, 313 Spl. Independentei, RO-060042 Bucharest, Romania. Tel.: +40 214029102; fax: +40 214029120.

E-mail address: cstan@physics.pub.ro (C. Stan).

0378-4371/$ – see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2009.08.005

The analysed data represent daily exchange rates of the national currencies with respect to the United States Dollar (USD) acquired from electronic sites [9]. The USD is chosen as the reference for the following reasons: it is the currency of a powerful economy and it was stable over the analyzed interval; the availability of the data was also an important factor. In order to get long enough samples as required for a reliable processing, we selected countries exhibiting a certain stability of their economies in the interval from 1 January 1996 until 31 December 2006. We focus on 20 economic systems that, during this period, did not change currency, did not significantly change their IEF and had an IEF score averaged over the 1996–2006 interval above 50. The exchange rate time series discussed consists of $N = 3957$ data.

We obtain the same results if the analysis is carried out using the price changes (derivatives of the exchange rate series)

$$z_n = x_{n+1} - x_n \quad (n = 1, 2, \ldots, N-1), \tag{1}$$

where the individual data are denoted x_n $(n = 1, 2, \ldots, N)$, or the returns

$$r_n = (x_{n+1} - x_n)/x_n \quad (n = 1, 2, \ldots, N-1). \tag{2}$$

3. The box and whisker plot

The distribution of data in a series can be estimated in many ways. The box and whisker plot represents a simplified alternative to the histogram [10,11]. The first step is the identification of the quartiles (Q_1, Q_2, Q_3). The box and whisker diagram is also called a five-point plot because, in addition to the lower (Q_1) and upper (Q_3) quartiles it uses three other characteristics of the series: the minimum, the median and the maximum, as shown in Fig. 1. The left and right extremities of a horizontal segment correspond to the minimum and the maximum value in the series, respectively. The position of the three quartiles Q_1, Q_2 (the median) and Q_3 are marked by points on the segment in the appropriate positions. Then, two boxes are drawn: one between the lower quartile and the median and another one between the median and the upper quartile. The sub-segment between the left box and the minimum point and the sub-segment between the right box and the maximum point represent the whiskers.

A box and whisker plot is useful in the description of the data because, at one glance, one can judge the symmetry or the skewness of the distribution. As a quantitative measure we consider the slimness of the distribution, defined as the ratio

$$S = \frac{R}{\Delta Q} \tag{3}$$

where $\Delta Q = Q_3 - Q_1$ is the interquartile interval and $R =$ Max–Min is the range. This ratio shows the relative magnitude of the middle 50% of the data. In the case of a Gaussian distribution $N(0, \sigma^2)$, the interquartile range is in the interval $(-0.6745\sigma, 0.6745\sigma)$. If the range is chosen between the lower and upper 0.1% probability bounds that correspond to the interval $(-3.09\sigma, 3.09\sigma)$, the slimness of a Gaussian is $S \approx 4.58$. For very long experimental or computer generated series of white Gaussian noise, S can be close to 5.

The slimness is very convenient for the characterization of leptokurtic and extremely leptokurtic distributions (fat tailed distributions) such as the ones we are dealing with. Various measures for fat tail characterization have been proposed, a relevant one being the tail index [12]. An important aspect to be taken into consideration in the case of fat tailed distributions is the problem of outliers. For the type of time series used in this study no outliers appear. This aspect has been extensively discussed and reliably explained by Stanley [13,14].

The distribution of the returns for all the exchange rate time series studied have S values considerably larger than the Gaussian noise. The problem of the origin of fat tails of financial time series is well documented, and highly competent discussions on the subject are available [1].

4. Iterated function systems (IFS) and the chaos game

IFS algorithms are used to computer generate fractal patterns. In the type of fractals of interest to this work, a rule from a set is applied to an initial point and a new point is generated. This becomes the initial point for the next application of the same rule or of another one from the set. The plotting of the multitude of these points represents the fractal [15,16]. To each rule a probability is associated, and the frequency of the application of a particular rule is proportional to the corresponding probability, but the order of application is random.

Here, we are interested in a particular type of IFS system called *the chaos game* [17,18]. It is "played" as follows. Consider a number of points in a plane representing the vertices of a polygon. Most often, the preferred number of points is four, representing the vertices of a square. To obtain the Sierpinski gasket, only three of the vertices are used. Suppose the three points considered are numbered in a clockwise direction, and number 1 is the lower right vertex.

Take an arbitrary point inside the triangle as *the seed*. At random, generate one of the three numbers, 1, 2, 3. If 2 comes out, then halfway between the seed and the vertex number 2 a point is plotted and the seed is erased. If the next number to come out is 1, then a point is plotted at mid-distance between the previously generated one and vertex number 1, and again the previous point is erased. This process with erasing previous points is continued for a few (say 10) steps because, if the initial point is not on the fractal (as usually is the case) then the following few points will also be relatively off the gasket.