

An interacting-agent model of financial markets from the viewpoint of nonextensive statistical mechanics

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Abstract

In this paper we present an interacting-agent model of stock markets. We describe a stock market through an Ising-like model in order to formulate the tendency of traders to be influenced by the other traders' investment attitudes [Kaizoji, *Physica A* 287 (2000) 493], and formulate the traders' decision-making regarding investment as the maximum entropy principle for nonextensive entropy [C. Tsallis, *J. Stat. Phys.* 52 (1988) 479]. We demonstrate that the equilibrium probability distribution function of the traders' investment attitude is the *q-exponential distribution*. We also show that the power-law distribution of the volatility of price fluctuations, which is often demonstrated in empirical studies can be explained naturally by our model which originates in the collective crowd behavior of many interacting-agents.

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1. Introduction

In the past decades, *the efficient market hypothesis* is the dominating paradigm in finance and financial engineering [1]. The efficient market hypothesis argues that the current price already contains all information and past prices can not be of help in predicting future prices. In an efficient market, stock prices are completely determined by its fundamentals, given by the present discounted value of the stream of future dividends. The prices would be driven solely by economic news (exogenous random shocks) regarding changes in fundamentals, so that prices follow a random walk [1]. Although numerous attempts have been made by economists to demonstrate the efficient market hypothesis, the authenticity of this hypothesis remains uncertain [2].¹ A new approach proposed as an alternative to the efficient market theory is the interacting-agent approach that models the trading process with its interaction on a large ensemble of heterogeneous traders. An emerging literature on interacting-agent dynamics has been initiated by Brock [4], Kirman [5], Aoki [6], and Lux [7]. A number of interacting-agent models proposed recently showed that they can generate

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¹We may say that the efficient market hypothesis lost ground rapidly following the accumulation of evidence against it. For instance, Shiller [3] finds that stock market volatility is far greater than could be justified by changes in dividends.

some ubiquitous characteristics, for example the clustered volatility and the scaling behaviors of price fluctuations, found for empirical financial data as a result of interactions between agents. It seems reasonable to posit that the emergence of realistic scaling laws based on the agents' interaction would lend convincing evidence in favor of the *Interacting-Agent Hypothesis*. Although the interacting-agent models are advocated as an alternative approach to the efficient market hypothesis which is equivalent to the *rational expectation hypothesis* in economics [8], little attention has been given to how probabilistic rules, in which an agent switches his opinion, are connected with an agent's expectation formation. Our previous paper [9] proposed a new expectation formation hypothesis, that is, *the relative expectation formation hypothesis* corresponding to the interacting-agent hypothesis. The relative expectations formation of interacting agents has been formalized by using the *minimum average energy principle for Boltzmann–Gibbs entropy* [10,11]. The aim of the present paper is to generalize the formulation of the relative expectation formation from the viewpoint of nonextensive statistical mechanics. Nonextensive statistical mechanics has been introduced as a generalization of the traditional Boltzmann–Gibbs statistical mechanics by Tsallis [12].² We shall present an interacting-agent model that follows the line of an Ising-like model of financial markets proposed by our previous work [9], and formulate the agents' decision-making regarding investment as the *maximum entropy principle for Tsallis entropy*. We also demonstrate that an equilibrium probability distribution of the agent's investment attitude, which is obtained as the so-called *q-exponential distribution*, can be derived from the relative expectations formation. We also show that the interacting-agent model provides a convincing explanation of an universal statistical characteristics of financial data, that is, the power-law tails of the distribution of volatility, measured by the absolute value of relative price changes [20].

2. An interacting-agent model

2.1. The relative expectation formation

We think of the stock market as a venue in which large numbers of traders who participate in stock trading. Traders are indexed by $i = 1, 2, \dots, N$. Traders invest their money in a stock, for example, a market index traded at a price at time t . They can either buy a stock or sell a stock. The trader's investment attitude x_i is defined as follows: if trader i is the buyer of a stock during a period, then $x_i = +1$, and if trader i , in contrast, is the seller of a stock during a period, then $x_i = -1$. A trader, who expects a certain exchange profit through trading, will attempt to predict every other trader's behavior in order to forecast the future movement of the price, and will choose either the same behavior or behavior contrary to the other traders' behavior. The trader's decision-making will also be influenced by changes in the news relating to the stock. When a trader hears good news about a stock, a trader may think that now is the time for him to buy the stock. Formally let us assume that the investment attitude of trader i is determined by minimization of the following evaluation function $e_i(x)$,

$$e_i(x) = -J \sum_{j=1}^N x_j x_i - b s x_i, \quad (1)$$

where J denotes the strength of the other traders' influence on the trader, and b denotes the strength of the reaction of a trader to the news s which may be interpreted as an external field, and x denotes the vector of investment attitude of all the traders $x = (x_1, x_2, \dots, x_N)$. A trader who has the positive value of J is called a *trend follower* and a trader who has the negative value of J is called a *contrarian*. The optimization problem that should be solved for every trader to achieve minimization of their evaluation functions $e_i(x)$ is formalized by the minimization of the following function,

$$E(x) = -J \sum_{i=1}^N \sum_{j=1}^N x_i x_j - \sum_{i=1}^N b s x_i. \quad (2)$$

²For applications of Tsallis statistics to economics, see Refs. [17–19].

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