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Modified local one-dimensional beam-propagation method based on the Douglas scheme

Wei-Ping Zang, Jian-Guo Tian *, Zhi-Bo Liu [1], Wen-Yuan Zhou, Feng Song, Chun-Ping Zhang

The MOE Key Laboratory of Advanced Technique and Fabrication for Weak-Light Nonlinear Photonics Materials, Nankai University, Tianjin 300457, China
Tianjin Key Laboratory of Photonics Materials and Technology for Information Science, Nankai University, Tianjin 300457, China

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Abstract

A finite-difference algorithm that combines the local one-dimensional approximation and the Douglas scheme is suggested to solve the three-dimensional non-linear Schrödinger equation. The truncation error in this scheme is reduced to $O(\Delta x)^4$ in the transverse direction, whereas that in the conventional local one-dimensional approximation is $O(\Delta x)^2$. Using this scheme, the computational accuracy and computational efficiency can be improved drastically without the introduction of additional programming works and computational time. Numerical results obtained for an elliptic Gaussian beam propagating in non-linear medium verify the advantages of this method.

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Keywords: Beam-propagation method; Local one-dimensional approximation; Douglas scheme

1. Introduction

An astigmatic Gaussian beam such as a mode-locked laser beam, a frequency-doubled laser beam, or a semiconductor laser beam, especially a beam from a Gaussian beam focused by a cylindrical lens or a beam from an astigmatic cavity, is often encountered practically. The analysis methods for this kind of optical beam propagating in non-linear media can be divided into three types. First type is conventional aberrationless constant shape approximation or variational method [1–4]. The difference between a simple aberrationless method and a variational method is that the latter

* Corresponding author.
E-mail addresses: jjtian@nankai.edu.cn (J.-G. Tian), rainingstar@eyou.com (Z.-B. Liu).
[1] Present address: Photonics Center, College of Physics, Nankai University, Tianjin 300071, China. Tel.: +86 2223508379; fax: +86 2223504856.

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optimizes both the beam-waist radius and the curvature radius and can give a better quantitative result for moderate non-linearity. This method is analytical one and does not always give accurate results for larger optical non-linearities. Second type is FFT-BPM (fast Fourier transform beam propagation method) algorithm [3,5], it can analyze numerically the elliptic Gaussian beam propagation in non-linear medium. But FFT-BPM algorithm has the following disadvantages due to the nature of the FFT [6]: (1) it requires a long computational time; (2) the propagation step has to be small; (3) the number of sampling points must be a power of two, and so on. Third type is FD-BPM (finite difference beam propagation method), it is well established for the analysis and simulation of beam propagation in non-linear medium with unconditional stability and good accuracy [7,8].

We have analyzed the elliptic Gaussian beam propagation in Kerr medium by a combination of the local one-dimensional (LOD) approximation and the Crank–Nicholson (CN) scheme [8]. That scheme utilizes a tridiagonal solution of one-dimensional situation, and is unconditionally stable and accurate to second-order in transverse direction and propagation direction, since it uses the Crank–Nicolson algorithm [7].

In this paper, a finite-difference algorithm that combines the local one-dimensional approximation and the Douglas scheme is suggested to solve the three-dimensional non-linear Schrödinger equation. The truncation error in this scheme is reduced to $O(\Delta x)^4$ in the transverse direction, whereas that in the conventional local one-dimensional approximation is $O(\Delta x)^2$. Numerical results obtained for an elliptic Gaussian beam propagating in non-linear medium verify the advantages of this scheme.

2. Methods

For simplicity, we call the scheme that combines the local one-dimensional (LOD) approximation and the Crank–Nicholson (CN) scheme as LOD_C scheme, the scheme with Douglas algorithm as LOD_D scheme.

If we write the field envelop $E = E_0\psi$, where E_0 is the field amplitude at $(z, x, y) = (0, 0, 0)$, and utilize the slow varying envelope approximation, the partial differential equation that describes the propagation of the electric field envelope in a non-linear medium can be written as

$$\frac{\partial\psi}{\partial\bar{z}} = \frac{-\mathrm{i}}{4}\left(\frac{\partial^2\psi}{\partial\bar{x}^2} + \frac{\partial^2\psi}{\partial\bar{y}^2}\right) - \mathrm{i}\cdot p(\psi)\psi, \tag{1}$$

where the transverse coordinates are normalized to the beam waist radius in x-direction, $(\bar{x}, \bar{y}) = (x/w_{0x}, y/w_{0x})$, the longitudinal coordinate is normalized to the Rayleigh length $\bar{z} = z/z_0, z_0 = kw_{0x}^2/2$, k is wave-vector, and $p(\psi)$ is a non-linear coupling function.

Let $\psi^r(\bar{x}, \bar{y})$ be the solution to Eq. (1) at $\bar{z} = \bar{z}^r$, then the solution at $\bar{z}^r = \bar{z}^r + \Delta\bar{z}$ can be written in terms of $\psi^r(\bar{x}, \bar{y})$

$$\psi^{r+1} = \exp\left[\frac{-\mathrm{i}\Delta\bar{z}}{4}\left(\frac{\partial^2}{\partial\bar{x}^2} + \frac{\partial^2}{\partial\bar{y}^2}\right) - \mathrm{i}\int_{\bar{z}^r}^{\bar{z}^{r+1}} p(\psi)\,\mathrm{d}\bar{z}\right]\psi^r. \tag{2}$$

This equation can be rewritten into a second-order accuracy form by utilizing split operator

$$\psi^{r+1} \approx \exp\left[\frac{-\mathrm{i}\Delta\bar{z}}{4}\left(\frac{\partial^2}{\partial\bar{x}^2}\right)\right]\exp\left[-\mathrm{i}\cdot\bar{p}(\psi)\Delta\bar{z}\right] \times \exp\left[\frac{-\mathrm{i}\Delta\bar{z}}{4}\left(\frac{\partial^2}{\partial\bar{y}^2}\right)\right]\psi^r, \tag{3a}$$

$$\bar{p}(\psi) = \frac{1}{\Delta\bar{z}}\int_{z^r}^{z^{r+1}} p(\psi)\,\mathrm{d}\bar{z}. \tag{3b}$$

The symmetrization of Eq. (3) in form is actually very important, since after the first upgrading of the phase the half steps of propagation in x- and y-direction can be combined into single propagation steps according to the rule

$$\exp\left[\frac{-\mathrm{i}\Delta\bar{z}}{4}\left(\frac{\partial^2}{\partial\bar{y}^2}\right)\right]\exp\left[\frac{-\mathrm{i}\Delta\bar{z}}{4}\left(\frac{\partial^2}{\partial\bar{x}^2}\right)\right] = \exp\left[\frac{-\mathrm{i}\Delta\bar{z}}{4}\left(\frac{\partial^2}{\partial\bar{y}^2} + \frac{\partial^2}{\partial\bar{x}^2}\right)\right]. \tag{4}$$

The algorithm for the field propagating over a distance $\Delta\bar{z}$ consists of the phase change caused by the non-linearity in medium, followed by a vacuum propagation of the resulting field over a distance $\Delta\bar{z}$ that complies with the equation