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Optics Communications 256 (2005) 451–469

OPTICS COMMUNICATIONS

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Linear and nonlinear propagation properties of Cosine-Gauss (CG) pulses in optical fibers

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Received 23 March 2005; accepted 28 June 2005

Abstract

From a recent interpretation [A. Gajadharsingh, P.-A. Bélanger, Opt. Commun. 241 (2004) 377] of the formation of a dispersion managed soliton (DMS) in the zero-average dispersion regime, we have seen that the use of CG pulses to approximate such solutions proves to be very accurate in this regime. Hence the basic linear and nonlinear propagation properties of such distributions deserve a closer look and are one of the subjects of this paper. We study, through second-order moment (SOM) theory, the root-mean-square (RMS) properties of a general CG ansatz and discuss the possible applications of such distributions. We also derive, using a perturbative approach, new approximate propagation laws through the moment theory which prove to be very accurate when compared to exact numerical results therefore providing us with analytical tools that can be used for various design purposes.

PACS: 42.81; 42.81.D; 42.65.T; 42.79.S

Keywords: Pulse propagation; Optical solitons; Dispersion management

1. Introduction

Modeling linear and nonlinear pulse and beam propagation is still one of the most studied and interesting topics nowadays. There exist various methods that are commonly used to study light propagation in various media and in various situations. The variationnal approach (VA) [2–5] is probably the most famous of all those techniques and has been widely used for beam and pulse propagation in all types of media and in 1+1D and 2+1D. Other approximate techniques, among others, involve ABCD matrices [6,7], perturbative

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0030-4018/$ - see front matter
doi:10.1016/j.optcom.2005.06.084

approach [8], multiple-scale expansion [9,10], direct scattering method [11] or SOM [12,13] which is based on the RMS properties [14] associated with the distributions studied. The moment theory is an attractive technique since it allows us to easily follow the gross characteristics of a pulse or beam submitted to various effects and will therefore be the approach that we shall use throughout this paper.

When studying some problem related to beam or pulse propagation, it is often required to approximate the propagating field with some accurate analytic expression so that various design purposes or theoretical predictions can be achieved. These are the basics of beam and pulse modeling and normally, commonly used distributions are the Gaussian and hyperbolic secant functions which are used as initial guess when studying pulse or beam propagation since Gaussian distributions can be easily generated experimentally and are the natural solutions of the linear problem whereas the hyperbolic secant distribution corresponds to the fundamental solitonic solution of the nonlinear Schrödinger equation (NLS). These two forms have also the advantage of being easy to use analytically although the use of hyperbolic secant distribution can be tricky at times. Recently, the interference of chirped complex conjugate pulses [1] in an optical fiber has been suggested as a way of interpreting the formation of DMS in the zero-average dispersion regime. From this work, the CG pulse distribution has been introduced and has been shown to be useful and more accurate whenever multiple side-lobes are present. Therefore the basic RMS, linear and nonlinear propagation properties as well as possible applications of such distributions deserve a closer look and are the subject of this paper.

Hence the paper is arranged as follows. We shall start by studying the basic RMS propagation properties of the most general of the CG distributions. Analytical expressions of the spectral and temporal variances for such distributions will be given and the shape of a few CG distributions will be illustrated in both the spectral and temporal domains. The linear propagation properties of such pulses will be closely studied by the derivation of the propagation laws for the temporal variance and the chirp associated with these pulses. Having done this, the nonlinear propagation laws will be derived for all the RMS parameters associated with the CG pulse by making use of a perturbative approach. It will be shown that these propagation laws are more accurate than those derived previously while being as accurate, easier to use and more general than those that can be deduced through the usual VA. The main advantages of the SOM method will be pointed out and finally some applications of those pulses will be discussed in the final section.

2. The CG distribution

In the spectral domain, a real CG pulse is given by:

$$\tilde{V}(\omega) = \tilde{A}\mathrm{e}^{-\omega^2 q_0^2}\cos[q_0^2\omega^2\tan(\phi_0) - \phi], \tag{1}$$

where q_0 is the width of the pure Gaussian pulse whereas ϕ_0 and ϕ are arbitrary phase factors. Taking the inverse Fourier transform of Eq. (1), we deduce the temporal distribution of the CG pulse:

$$V(t) = A\mathrm{e}^{-\frac{t^2\cos^2(\phi_0)}{4q_0^2}}\cos\left[\frac{\sin(2\phi_0)}{8q_0^2}t^2 + \left(\phi - \frac{\phi_0}{2}\right)\right]. \tag{2}$$

We note that the case for which $\phi = \phi_0 = 0$ corresponds to the simple Gaussian pulse and that in this case q_0 is the RMS width of the Gaussian distribution. Fig. 1 shows the general shape of the CG distribution for various choices of ϕ and ϕ_0 in the temporal (Fig. 1(a)) and spectral (Fig. 1(b)) domains, respectively.

We can clearly see the presence of multiple side-lobes which is the typical signature of the cosine term modulating the Gaussian envelope and it has already been shown and explained [1] that this cosine term can be obtained through the interference of two chirped complex conjugate Gaussian pulses. The importance of the side-lobes depends on the value of ϕ_0 which is directly related to the way in which the individual Gaussian pulses interfere. Numerical radiationless solutions of propagating pulses with oscillating tails can be observed when one studies DM for average to strong