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New representation of n-mode squeezed state gained via n-partite entangled state ☆

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Abstract

By virtue of the n-partite entangled state, we extend the way of Agarwal–Simon's presenting single-mode squeezed state to n-mode case and find a new representation of the n-mode squeezed state. This n-mode squeezed state is also an entangled state and can be a superposition of n-mode coherent states.

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Squeezed states have been important topic since 1970s due to their wide applications in optical communication and precise measurement in quantum optics [1]. They exhibit interesting nonclassical behavior. Many attempts have been made to find new squeezed states and new form of squeezing operators so that new experimental implementation could be proposed [2,3]. For the single-mode squeezed state wave function $\Psi(x,\sigma_x) = [1/(\pi\sigma_x^2)^{1/4}]\exp(-x^2/2\sigma_x^2)$ (its wave packet's width is $\sigma_x^2/2$) Agarwal and Simon found a new representation [4]

$$|\Psi\rangle_x \equiv s_x^{1/2}\exp\left[-\frac{1}{2}(s_x^2-1)X_1^2\right]|0\rangle \quad s_x^2 = 1/\sigma_x^2, \tag{1}$$

or for $\Psi(p,\sigma_p) = [1/(\pi\sigma_p^2)^{1/4}]\exp(-p^2/2\sigma_p^2)$,

$$|\Psi\rangle_p \equiv s_p^{1/2}\exp\left[-\frac{1}{2}(s_p^2-1)P_1^2\right]|0\rangle \quad s_p^2 = 1/\sigma_p^2, \tag{2}$$

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where X_1 and P_1 are the coordinates and momentum operators, respectively. Let $f = -\frac{1}{2}(s_p^2 - 1) \quad s_p = e^{\lambda}$, we have the single-mode squeezing operator,

$$S_1 \equiv (1-2f)^{1/2} e^{fP_1^2}. \tag{3}$$

Using S_1, Eq. (2) can be expressed as

$$|\Psi\rangle_p \equiv (1-2f)^{1/2} e^{fP_1^2}|0\rangle = \operatorname{sech}^{1/2}\lambda \exp\left[\frac{1}{2}a_1^{\dagger 2}\tanh\lambda\right]|0\rangle, \tag{4}$$

which is just the traditional standard form of the single-mode squeezed vacuum state. Moreover, Agarwal–Simon's representation shows that the single-mode squeezed state can be a superposition of coherent states on a line in phase space, i.e.,

$$|\Psi\rangle_p = \left(\frac{s_p}{\pi(s_p^2-1)}\right)^{1/2} \int_{-\infty}^{\infty} dx \exp\left(-\frac{x^2}{s_p^2-1}\right)|x\rangle_{\rm coh}, \tag{5}$$

where $|x\rangle_{\rm coh} = \exp(-i\sqrt{2}P_1 x)|0\rangle$ is coherent state [5,6]. Enlightened by Agarwal–Simon's idea and employing the two-mode entangled state, in [7] we have derived a new form of the two-mode squeezing operator

$$S_2 \equiv (1-2f)^{1/2} e^{f\left[(X_1-X_2)^2+(P_1+P_2)^2\right]/2}, \tag{6}$$

whose normally product form is

$$S_2 = (1-2f)^{1/2}\frac{1}{1-f} : \exp\left[\frac{f}{1-f}\left(a_1^\dagger - a_2\right)\left(a_1 - a_2^\dagger\right)\right]. \tag{7}$$

Operating S_2 on the vacuum state $|00\rangle$ will yield the two-mode squeezed state

$$S_2|00\rangle = \operatorname{sech}\lambda \exp[a_1^\dagger a_2^\dagger \tanh\lambda]|00\rangle, \tag{8}$$

which is just the traditional standard form of the two mode squeezed vacuum state. An interesting problem thus naturally arises: Can Agarwal–Simon's idea be extended to n-mode case? We will deal with this problem in the article. Firstly, we need introduce a n-mode entangled state with continuous variables. To derive compact result, we choose the Jacobian coordinates used in traditional many-body theory [8] as observable variables (One of the advantages of using Jacobian coordinates is that the motion of center-of-mass can be separated off from the relative motion of many particles). Noting that for n-partite system the Jacobian coordinates operators $X_j - \frac{1}{j-1}\sum_{k=1}^{j-1} X_k \quad (j = 2,\ldots,n)$ and the total momentum operator $\sum_{i=1}^{n} P_i$ are permutable with each other, we can introduce the common eigenstate of them [9]

$$\begin{aligned} |p\chi_2\chi_3\cdots\chi_n\rangle = \frac{1}{\sqrt{n}\pi^{n/4}} \exp\Bigg[& -\sum_{j=2}^{n}\frac{1}{2j}(j-1)\chi_j^2 - \frac{1}{2n}p^2 + \frac{\sqrt{2}ip}{n}\sum_{j=1}^{n}a_j^\dagger + \sqrt{2}\sum_{j=1}^{n}\chi_j a_j^\dagger \\ & -\sqrt{2}\sum_{j=1}^{n}\left(\frac{\chi_j}{j}\sum_{k=1}^{j}a_k^\dagger\right) + \frac{2}{n}\sum_{j\prec k;j,k=1}^{n} a_j^\dagger a_k^\dagger + \left(\frac{1}{n}-\frac{1}{2}\right)\sum_{j=1}^{n}a_j^{\dagger 2}\Bigg]|0\cdots 0\rangle, \end{aligned} \tag{9}$$

where $\chi_1 = 0$ which obeys the eigenvector equations

$$\left(\sum_{i=1}^{n}P_i\right)|p\chi_2\chi_3\cdots\chi_n\rangle = p|p\chi_2\chi_3\cdots\chi_n\rangle \tag{10}$$

and