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# Optical image encryption based on fractional wavelet transform

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**Abstract**

Based on the fractional wavelet transform (FWT) a novel method for the image encryption is proposed. We encrypt the image by two fractional orders and a series of scaling factors. There are two series of keys in this method. Only when all of these keys are correct, the image could be decrypted. We can also realize the partial encryption by using this method. Meanwhile, the optical implementation is suggested and some numerical simulations prove its possibility.


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*Keywords:* Image encryption; Decryption; Fractional wavelet transform; Fractional fourier transform

## 1. Introduction

In order to protect people's privacy, information security problem has been paid more and more attention with the development of the science and technology. In the past decade, a lot of work has been done on the security of information, and quite many optical image encryption methods were presented [1–11]. Some of the encryptions are based on the fractional Fourier transform (FRT) [1–8], the fractional orders are the additional keys compared with the traditional Fourier transform. Among them, some techniques secure and store the information by multiplying random phases in the input planes or in some fractional domains [2–7], and the digital holography is also used to encrypt the image [8]. Some other methods are proposed by using the polarization, coherence properties of the light to realize the encryption [9,10]. All these encryptions are performed either in digital computational forms or in optical experimental forms. The digital computational technique is based on

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the concept of the optical experiment that is called the virtual-optics system [11]. CCD cameras are used to record the encryption results. Meanwhile, the corresponding decryption methods are also given in these methods.

We know that the wavelet transform (WT) has been applied widely in the signal processing, because it can successfully analyze and process non-stationary signals compared with the ordinary Fourier transform. The WT has also been applied effectively in the image encryption and compression in the network, multimedia and communications [12]. Much work has been done on the image encryption with WT. It can also realize the partial encryption by the data decomposition and hiding the important parts. Two methods were proposed for the partial encryption based on the quadtree compression and zerotree wavelet image compression [13].

The FWT was first defined by Mendlovic and Zalevsky [14] in 1997. And the optical realization of the WT for two-dimensional objects was also proposed by Mendlovic and Kontorti [15] in 1993. Therefore, it is possible for us to realize the optical image encryption based on the FWT. In this paper, we provide a novel method to encrypt and decrypt the image based on the FWT. It is better than the encryptions based on FRT and WT and can join the virtues of FRT and WT. The fractional orders of FRT and scaling factors of WT are the important keys. The keys can be randomly chosen, and the unauthorized people can not obtain information without the correct keys, so the image can be well protected. This method can be used both in optical encryption and wireless signal processing. And partial encryption technique can also be realized by this method. The partial encryption is also called selected encryption that can be realized by wavelet decomposition or wavelet packet decomposition. It is useful in signal processing and wireless data communication.

## 2. Basic theory

As we know, the two-dimensional (2-D) FRT is defined as [16–18]:

$$F^{\phi_1,\phi_2}[f(x,y)](x',y') = \int\int B_{p_1,p_2}(x.y;x',y')f(x,y)\,\mathrm{d}x\,\mathrm{d}y, \tag{1}$$

with the kernel

$$B_{p_1,p_2}(x,y;x',y') = B_{p_1}(x,x')B_{p_2}(y,y'), \tag{2}$$

and

$$B_{p_1}(x,x') = \frac{\exp\{-\mathrm{i}[\pi\,\mathrm{sgn}(\sin\phi_1)/4-\phi_1/2]\}}{|\lambda f_{s1}\sin\phi_1|^{1/2}} \times \exp\left(\mathrm{i}\pi\frac{x^2+x'^{,2}}{\lambda f_{s1}\tan\phi_1} - 2\pi\mathrm{i}\frac{xx'}{\lambda f_{s1}\sin\phi_1}\right), \tag{3}$$

$$B_{p_2}(y,y') = \frac{\exp\{-\mathrm{i}[\pi\,\mathrm{sgn}(\sin\phi_2)/4-\phi_2/2]\}}{|\lambda f_{s2}\sin\phi_2|^{1/2}} \times \exp\left(\mathrm{i}\pi\frac{y^2+y'^{,2}}{\lambda f_{s2}\tan\phi_2} - 2\pi\mathrm{i}\frac{yy'}{\lambda f_{s2}\sin\phi_2}\right), \tag{4}$$

where $\phi_1 = p_1\pi/2$ and $\phi_2 = p_2\pi/2$, $\phi_1$ and $\phi_2$ correspond to the rotating angles in Wigner domains, $p_1$ and $p_2$ are the fractional orders. When $p_1 = p_2 = 1$, Eq. (1) corresponds to the conventional Fourier transform.

And the 2-D hybrid FWT of the signal $f(x,y)$ can be defined as follows:

$$W(a_{mn},\bar{b}) = \int\int\int\int B_{p_1,p_2}(x,y;x',y')f(x,y) \times h^*_{a_{mn}\bar{b}}(x',y')\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}x'\,\mathrm{d}y'. \tag{5}$$

When $p_1 = p_2 = 1$, it reduces to the conventional WT. And in the fractional domains it can be written as

$$W(a_{mn},\bar{b}) = (a_m a_n)^{1/2}\int\int H^*(a_m u, a_n v) \times \exp(\mathrm{j}2\pi u b_{x'},\mathrm{j}2\pi v b_{y'}) \times F\left\{F^{\phi_1,\phi_2}[f(x,y)](x',y')\right\}(u,v)\,\mathrm{d}u\,\mathrm{d}v, \tag{6}$$

while its back-reconstructing formula in the fractional domains is

$$f(x,y) = \frac{1}{C}\int\int F\left\{\sum_m\sum_n\int\int\frac{1}{a_m a_n}W(a_{mn},\bar{b}) \times H(a_m u, a_n v)\exp(-\mathrm{j}2\pi u b_{x'},-\mathrm{j}2\pi v b_{y'}) \times \mathrm{d}b_{x'}\,\mathrm{d}b_{y'}\right\}(x',y')B_{-p_1,-p_2}(x,y;x',y')\,\mathrm{d}x'\,\mathrm{d}y', \tag{7}$$