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# Tunneling among rotation tori

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ABSTRACT

We consider an integrable Hamiltonian system generated by the resonant normal form in order to study a particular mechanism of tunneling. We isolated near doublets of energy corresponding to rotation tori of the classical dynamics counterpart and the degeneracies breakdown is attributed to rotation–rotation tunneling.

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## 1. Introduction

Tunneling effects in dynamical systems have been studied using different approaches in the last years. The standard model, in order to understand tunneling, is the one-dimensional double well potential where a quantum particle with energy smaller than the energy of the top of the barrier can tunnel from one well to the other. The corresponding phase space presents a separatrix whose manifolds emanate from, and arrive in, the unstable hyperbolic fixed point associated with the top of the barrier. Inside the separatrix there are libration tori with lower energies and in the minima of the wells there are two stable elliptic fixed points. Then, for this model, a simple visualization of the tunneling effect in the phase space is a particle jumping from one libration torus into a homoclinic loop to other libration torus into the other loop. These homoclinic loops correspond to the left and right sides of the separatrix, see Ref. [1] for an initial reading. Thus, tunneling occurs in a range of energies which are very close of the separatrix energy. So, since it is established an association between separatrix and tunneling, it sounds natural to talk about tunneling every time that a separatrix appears in any integrable dynamical system. On the other hand, in chaotic systems the separatrices are destroyed but it is still possible to have tunneling effects involving quantum states supported in resonance islands or, in a more advanced fashion, tunneling effects between congruent tori originated by some discrete symmetry which are called chaotic tunneling [2–17]. But in this paper we will focus on a particular mechanism of tunneling in an integrable dynamical system involving rotation tori. The model that we will use is a Hamiltonian expressed by the Birkhoff–Gustavson Normal Form [2,5,18,19], or simply Resonant Normal Form, because this paper is a natural sequence of our work presented in Ref. [20].

The paper is organized as follows, in Section 2 we present the system with the essential formulas and in Section 3 we discuss the numerical results with our conclusions.

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## 2. Describing the model

We will consider the Hamiltonian that we have studied previously [20] which consists in an integrable system represented by the expansion of the Resonant Normal Form in the neighborhood of a stable equilibrium point. A remarkable characteristic of this model is that we can construct a desired Hamiltonian from this expansion in order to approximate a real system or to generate a specific effect and because that one considers this expansion as a toy model. The considered system is autonomous with two degrees of freedom and its Hamiltonian is generated through a series expansion on the position/ momentum variables ($q_k$, $p_k$, $k = 1$ or 2), in such way that it has a first term corresponding to a 2-d harmonic oscillator and all non-linearities are enclosed in the series expansion. In order to prepare the prototype to study tunneling involving rotation tori we follow the strategy developed in Refs. [5,20], performing the following three canonical transformation (CT): (i) initially we describe the system in the complex variables ($a_k, a_k^*$) defined as, $a_k = \frac{q_k + \mathrm{i}p_k}{\sqrt{2}}$ and $a_k^* = \frac{q_k - \mathrm{i}p_k}{\sqrt{2}}$, and we truncate properly the infinite expansion; (ii) next we introduce the action/angle variables ($I_k, \varphi_k$) through $a_k = \sqrt{I_k}\exp(\mathrm{i}\varphi_k)$ and $a_k^* = \sqrt{I_k}\exp(-\mathrm{i}\varphi_k)$ where $I_k = \frac{p_k^2+q_k^2}{2}$, so that the three isochronous resonance of order 1:4 are visible; (iii) finally we perform the last CT to the new action/angles variables ($J_k, \theta_k$) defined through the equations, $I_1 = 4J_1$, $\theta_1 = 4\varphi_1 - \varphi_2$, $I_2 = J_2 - J_1$, $\theta_2 = \varphi_2$.

The system when described in ($q_k, p_k$) presents three necklace-like chains, with four islands in each chain, involving the stable equilibrium point. In the ($I_k, \varphi_k$) variables, the three chains are stretched presenting the four islands in the range $\varphi_1 : [0, 2\pi]$. The last CT makes a zoom in a single island and put it in the range $\theta_1 : [0, 2\pi]$.

The resulting Hamiltonian is then described in the action-angle variables ($J_k, \theta_k$) and it is decomposed in the following two terms, $H = H_0 + \alpha H_1$,

$$H(J_1, J_2, \theta_1) = \left\{ J_2 - \frac{a}{2}(4J_1 - c)^2 + \frac{1}{4}(4J_1 - c)^4 \right\} + \alpha \left[ b(4J_1 - c)^2 - a \right] (4J_1)^2 \sqrt{J_2 - J_1} \cos\theta_1 \tag{1}$$

where $H_0$ is the term in keys and $H_1$ is the other one. The parameters $a$, $b$, $c$ and $\alpha$ are adjustable. Due to the fact that the $\theta_2$ variable does not appear explicitly in the Hamiltonian, the $J_2$ action is a constant of motion. The term $H_0$ is called non-perturbed and it is given by a 4-th degree polynomial in the $J_1$ action, which allows the introduction of the three isochronous resonance chains in the system. The term $H_1$, called perturbation, is periodic in $\theta_1$ and it has a $J_1$-polynomial in the multiplicative pre-factor. This quadratic polynomial has two real roots, which means that when the system passes through any of these roots, the perturbation is algebraically null, independently of the value of the perturbation parameter, giving origin to two robust tori. These robust tori correspond to spanning curves which avoid that a trajectory passes from a region to another one separated by them. The periodic perturbation introduces the three isochronous resonances whose localizations in the phase space are defined by the non-perturbed term, that is, from the Hamilton's equations for the non-perturbed system we get the three values of the actions that locate the chains. They are,

$$\begin{aligned} J_{1+} &= \frac{c + \sqrt{a}}{4} \\ J_{1m} &= \frac{c}{4} \\ J_{1-} &= \frac{c - \sqrt{a}}{4} \end{aligned} \tag{2}$$

where the index $m$ is associated with the middle chain. On the other hand the roots of the pre-factor of the perturbation supply the localizations of both robust tori,

$$\begin{aligned} J_{1R+} &= \frac{c + \sqrt{a/b}}{4} \\ J_{1R-} &= \frac{c - \sqrt{a/b}}{4}. \end{aligned} \tag{3}$$

There are two distinct topological configurations, (i) one when the robust tori are intercalated with the resonance chains, which hinders the occurrence of Libration–Libration-like tunneling between distinct chains; (ii) another one when the robust tori are external to the chains of resonances. In the first case the chains are aligned, this means that the elliptical points of the different chains have the same values of $\theta_1$ (as well as the hyperbolic points). In the second case the chains are defocused, performing a "chess-like" scenario. The mechanism that generates the defocalization process is given by pitchfork-like bifurcations which occur on the robust tori when the separatrices overlap them [20].

The proposal of this work is to isolate numerically the effects of tunneling involving only rotation tori. In such way we have adjusted the parameters of the Hamiltonian in order to dislocate the central resonance chain in the energy scale, so that its energy is higher than the external ones. In the three-dimensional space ($\theta_1$, $J_1$, Energy) we have a similar geometrical configuration as the one of the double well (see Fig. 1 as a guide) where the external resonance chains appear in the minima of the wells while the central one appears in the maximum. Thus, the phase space corresponds to a projection of the chains on the plane ($\theta_1$, $J_1$) (Fig. 2).

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