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Superposition of monochromatic Bessel beams in (k_ρ, k_z)-plane to obtain wave focusing: Spatial localized waves

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Abstract

In this work we analyze the effect of superimposing monochromatic Bessel beams with different wave-vectors through the use of a multi-annular slit (or holographic methods). An analytical procedure based on the scalar diffraction theory allows one to obtain spatially localized waves.

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1. Introduction

After the pioneering work on Bessel beams by Durnin et al. in 1987 [1], non-diffracting beams have been drawing growing interest due to the fact that under ideal conditions they propagate without spreading [2–6]. These waves have potential applications in technology, like wireless communications, metrology, laser surgery, non-linear optics, optical tweezers, conduits in atom optics and so on [7–12].

Several solutions for the wave equation that are localized in spacetime have been obtained in the current literature and are best known as localized pulses [13–18], which propagate in free space without suffering diffraction and dispersion. Such solutions can be obtained by superposing diffraction-free beams in the frequency domain. As an example, one may invoke the X-shaped solutions, which are synthesized by an adequate superposition of Bessel beams in frequency domain.

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While non-diffracting beams are not localized axially, localized pulses propagate forward along the z-axis, as a function of $\zeta = z - vt$. Our purpose here is to obtain a well-localized wave in both the transverse and axial directions using a monochromatic source and standard optical elements of the kind employed by Durnin [1]. The possibility of transverse shape modeling using a superposition of Bessel beams was previously examined by Bouchal et al. while the control of longitudinal intensity pattern has been analyzed in Refs. [19] and [20] using numerical optimization techniques. In Ref. [21] the superposition of Bessel beams to obtain a stationary localized wave field of arbitrary longitudinal shape is demonstrated analytically using the Fourier series. The main difference between the localized pulses and our solutions is that in the former the peak's intensity travels along the z-axis with the time, while in our solution the peak remains static along the z-axis, i.e., the axial and transverse patterns are time independent. In other words, the X-shaped waves are obtained through a frequency superposition of Bessel beams with a defined unitary $\hat{k}$-vector (with $\hat{k} = \mathbf{k}/k$), which means that independently of the frequency $\omega = ck$ the $\mathbf{k}$-vectors belong to a cone with a fixed axicon angle θ, being $k_z = k\cos\theta$ and $k_\rho = k\sin\theta$. Our aim is to make an adequate superposition of Bessel beams with a fixed frequency ω_0 and wave number $k = \omega_0/c$ but with wave vectors $\mathbf{k}$ belonging to different cones and consequently, different angles θ. To do this a multi-annular slit with different radius of the rings can be used in an apparatus similar to that proposed in [1]. In fact, here we discuss the logarithmic spacing of the rings in the annular slit.

The remainder of this paper is described as follows: In the next section we develop the theoretical framework based on the scalar diffraction theory to analyze the multi-annular structure. In Section 3 we present and discuss a few results for the ideal case of a lens of infinite radius and in the last section, the conclusions and remarks are added.

2. Theoretical framework: multi-annular structures

At optical frequencies the scalar diffraction theory can be used to obtain excellent agreement with experiment [22]. Here, the scalar wave function Ψ is a possible representation of the electric field, E_x, or the magnetic field, H_y, being the Poynting's vector proportional to $|\Psi|^2$. The Kirchhoff's integral in the paraxial approximation (distant field or Fraunhoffer region) reads [22–25]

$$\Psi(\rho,\varphi,z) = \frac{k\exp\left[\mathrm{i}k\left(z+\frac{\rho^2}{2z}\right)\right]}{2\pi\mathrm{i}z} \times \int_0^\infty \varrho\,\mathrm{d}\varrho \int_0^{2\pi} \mathrm{d}\alpha\,\tau(\varrho,\alpha)\Psi(\varrho,\alpha,z'=0) \times \exp\left(\mathrm{i}\frac{k\varrho^2}{2z}\right)\exp\left[-\mathrm{i}\frac{k\rho\varrho}{z}\cos(\alpha-\varphi)\right], \tag{1}$$

where $k = 2\pi/\lambda$ is the vacuum wave number, $\tau(\varrho, \alpha)$ is the transmittance function (TF) of the aperture, $\Psi(\varrho, \alpha, z' = 0)$ is the incident wave, (ϱ, α, z') and (ρ, φ, z) are the diffractive aperture and observation points in cylindric coordinates, respectively.

Consider an ideal multi-annular slit with n concentric rings as shown in Fig. 1. The experimental setup is the same as that one proposed by Durnin [1] except for the slit. We assume that the condition

$$\delta a_j \ll \frac{\lambda f}{R} \tag{2}$$

holds, where a_j is the radius and δa_j is the thickness of the jth ring, λ is the wavelength of the laser illuminating the rings, f is the focal distance and R is

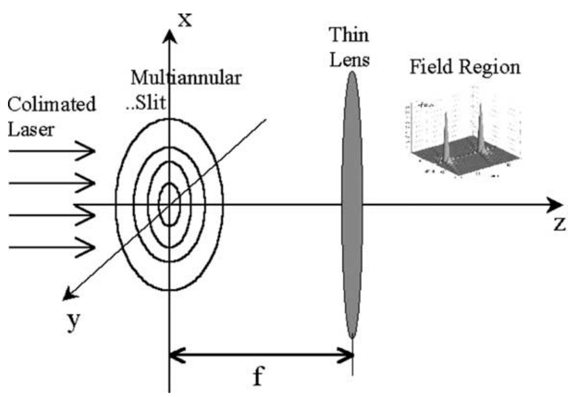

Fig. 1. Experimental setup with a multi-annular slit.