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Effect of imitation in evolutionary minority game on small-world networks

Jiale Chen, Hongjun Quan *

Department of Physics, South China University of Technology, Guangzhou, 510641, PR China

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ABSTRACT

The system performance in an evolutionary minority game with imitation on small-world networks is studied. Numerical results show that system performance positively correlates with the clustering coefficients. The domain structure of the agents' strategies can be used to give a qualitative explanation for it. We also find that the time series of the reduced variance σ^2/N could have a phasic evolution from a *metastable state* (two crowds are formed but the distribution of their probabilities does not peak at $p \approx 0$ and $p \approx 1$) to a *steadystate* (the two crowds evolve into a crowd and an anticrowd with the distribution of their probabilities peaking at $p \approx 0$ and $p \approx 1$).

1. Introduction

As an agent-based model of complex adaptive systems, the minority game model (MG) by Challet and Zhang [1] has attracted much attention among scientists in different research areas in the last ten years. In the MG, agents of an odd number N, each with S strategies and a memory size m, repeatedly compete to get into a minority group for limited resources in the market by adapting their strategies according to their experience. In every turn, agents always use their highest scoring one among his S strategies. Both numerical simulation and analytic research show that the distributions of agents' strategies and history are very important factors determining the systems performance [2]. It is found that unintentional cooperation among the intrinsically selfish agents can lead to a reduced waste of resources.

The evolutionary minority game proposed (EMG) by Johnson et al. [3] is an evolutionary version of MG in which all agents carry the same dynamical strategy at a moment in time together with a p-value (gene value), characterizing the probability that the agent is to follow the prediction of the strategy. At the end of each turn, agents who end up in the minority (majority) group win (lose) and are awarded (deducted) one point. Poorly performing agents with scores below a threshold d are allowed to change their p-value by taking a new value in an interval $[p - dp, p + dp]$. Upon strategy modification, the agent's score is reset to zero. The most interesting feature in EMG is that agents who behave in an extreme way (i.e., using $p \approx 0$ or 1, which corresponds to never or always following the prediction of the strategy) perform better than the cautious ones (i.e., using $p \approx 1/2$). This in turn leads to a self-segregation of the population in the sense that the distribution of p-values tends to peak at $p \approx 0$ and 1. Lo et al. gave an analytical explanation for this self-organized effect [4]. Subsequently, some modifications of EMG are proposed to investigate the factors for the self-segregation [5–10].

* Corresponding author. Tel.: +86 20 87114127; fax: +86 20 87112837.
E-mail address: hjquan@scut.edu.cn (H. Quan).

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In Refs. [8–10], the effect of imitation on EMG is studied. It was found that the segregation becomes more pronounced after introducing imitation. As a result, the system as a whole gets benefit from the imitation. However, in Refs. [11] Quan et al. found the system with imitation would take a phase transition as the prize-to-fine ratio R (defined as the ratio of the point awarded to the point deducted) turns from 1 to some value below $R_c (R_c < 1)$. When $R < R_c$ the imitation tends to increase the standard deviation compared with the origin EMG and is harmful for the market.

Complex networks are introduced recently to study the effect of imitation on EMG. In Ref.[12], Kirley considered the EMG on small-worlds networks and found the best performance of system occurred on a small-world network. Shang et al. [13] found that the dynamics of the system depends crucially on the structure of the underlying network. The strategy distribution in a star network is sensitive to the precise value of the mutation magnitude dp, in contrast to the strategy distribution in regular, random and scale-free networks, which is easily affected by the value of the prize-to-fine ratio R.

In this paper, we study further the effect of imitation between agents to EMG on small word networks and focus on the effects of the average path length L, the clustering coefficient C and the gene-value mutation magnitude dp on the variance. Our results show that there exists positive correlation between C and the system improvement. The domain structure of the agents' strategies is used to give a qualitative explanation. The system's performance for regular network could be better than that for small-world network, which is different from that in Ref. [12]. We also found that the time series of the reduced variance σ^2/N undergoes two phases during the evolution process, i.e., it first goes into a metastable phase where the population forms two crowds but the gene values does not peak at $p \approx 0$ and 1, and then goes into a stable phase where the gene values of the two crowds turn to peak at $p \approx 0$ and 1.

The plan of the paper is as follows. In Section 2, we define our model incorporating the effects of information transmission into EMG on network. Numerical results and discussions are presented in Section 3.

2. The model

We first use the original model proposed by Watts and Strogatz [14] to construct small-world networks, which are formed by the following random rewiring procedure.

Starting from a ring lattice with N vertices and K edges per vertex, we rewire each edge at random with the probability P_s. Tuning P_s from 0 to 1, the network would change from a regular one to a small-world one, then to a random one.

The structural properties of the networks with different P_s can be quantified by their characteristic path lengths $L\,(P_s)$ and clustering coefficients $C\,(P_s)$. Here L is the median of the means of the shortest path length connecting each vertex to all other vertices, which is defined as follow:

$$L = \frac{1}{N(N-1)} \sum_{i \neq j} d_{ij} \quad (1)$$

where d_{ij} is the shortest path length connecting vertex i and j, which can be simply defines as the number of edges along the shortest path between vertex i and j.

The clustering coefficient C is defined as follow:

$$C = \frac{1}{N} \sum_i c_i \quad (2)$$

where c_i is the clustering coefficient of agent i [14], for any given vertex i,

$$c_i = \frac{\text{(links between the vertices within its neighbourhood)}}{\text{(Number of links that could possibly exist between them)}}. \quad (3)$$

Thus L measures the typical separation between two vertices in the networks (a global property), whereas C measures the cliquishness of a typical neighborhood (a local property). L and C are both large for the regular lattice with $P_s = 0$; L and C are both small for the random network with $P_s = 1$; L is small but C is large for the small-world network.

Our model consists of an odd number N of agents, who are mapped into vertices on networks. All agents hold the same strategy, which is simply a record of the most recent trend. Each agent carries a probability p (p-value), characterizing the probability that the agent takes an action based on the prediction of the strategy. Each agent has to choose between two groups, 0 or 1, at each time step. The group with fewer agents is the winning group. The winners, i.e., those in the minority group, win one point, while the losers, i.e., those in the majority group, lose one point.

In addition to the aspects above, evolution in the present model is made through local information transmission among agents. As the agents are mapped into vertices on networks, an agent is called another's nearest neighbor if there is a link between them. Each agent has some nearest neighbors and he also knows the scores and the p-values of his neighbors. By this we can model situations in which groups of agents tend to share information. The best neighbor at the time step t is the one with the highest score. When several neighbors' scores are the highest at the same time, one of them is randomly chosen to be the best one.

If the agent's score falls below a value d ($d < 0$), and he has a lower score than his best neighbor, he will modify his p-value by choosing a new p-value randomly within a range $2dp$ centered on the p-value of his best neighbor; otherwise, he chooses a new p-value randomly within a range $2dp$ centered on the p-value of his old p. A reflective boundary condition is imposed in the p-space insuring that $0 \le p \le 1$ (i.e., if new $p < 0$, then let $p = |new p|$, if new $p > 1$, let $p = 1-(\text{new}\,p-1)$).