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Nonparaxial propagation analysis of elliptical Gaussian beams diffracted by a circular aperture

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Abstract

The direct integration of the diffraction integral is quite time consuming. Based on the fact that a hard-edge aperture function can be expanded into finite sum of complex Gaussian functions, a nonparaxial propagation expression for elliptical Gaussian beams diffracted by a circular aperture is derived using the well-known method of the scalar angular spectrum and the stationary phase. Simulation shows that when the *f*-parameter is greater and the truncation parameter is smaller, the paraxial approximation is invalid and the nonparaxial approach has to be used for apertured elliptical Gaussian beams. A circular aperture can cause the stigmatic elliptic Gaussian beam diverge in the far field but change the aspect ratio of the beam. It can also change the shape and intensity distribution of the higher-order Hermite–Gaussian beams due to the obstruction and the interference of beams.
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1. Introduction

The angular spectrum representation has been used to solve a variety of problem involving propagation, transmission, and reflection of Gaussian beams [1]. By using this method, Carter gave the electromagnetic field of a Gaussian beam with an elliptical cross-section [2], Agrawal and Pattanayak obtained the first-order correction of Gaussian beams [3], and Zeng et al. [4] presented the far-field expressions for off-axis Gaussian beams. Chen et al. [5] derived the propagation equations of vector Gaussian beam by using vector angular spectrum approach and compared the validity of paraxial and spherical approximations. Their results show that spherical approximation based on the method of stationary phase becomes more accurate for larger propagation distance.

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However, these earlier studies have been restricted to the unapertured propagation of Gaussian beams. In practice, the aperture effect exists more or less. Recently, Duan and Lü [6] presented the nonparaxial analysis of the far-field properties of circular Gaussian beams diffracted at a circular aperture. The present paper is aimed at studying the nonparaxial propagation properties of elliptical Gaussian beams diffracted by a circular aperture. In fact, the elliptical Gaussian beam represents the more general case, and the circular Gaussian beam can be regarded as its special case.

2. Analytical expression for apertured elliptical Gaussian beams

The field of an initial higher-order Hermite–Gaussian beam at the plane $z = 0$ is given, in the Cartesian coordinates systems, by [7]

$$E_0(x,y,0) = H_m\left(\sqrt{2}\frac{x}{w_{0x}}\right)H_n\left(\sqrt{2}\frac{y}{ew_{0x}}\right) \times \exp\left[-\left(\frac{x^2}{w_{0x}^2}+\frac{y^2}{e^2w_{0x}^2}\right)\right], \tag{1}$$

where $e = w_{0y}/w_{0x}$ is defined as the ellipticity, w_{0x} and w_{0y} are the waist widths in the x- and y-directions, respectively. We assume $w_{0x} \geqslant w_{0y}$ and the beam becomes the circular when $e = 1$. H_j is the jth order Hermite polynomial. Assuming that a circular aperture of radius a is placed at the plane $z = 0$, the field just behind the aperture reads as

$$E(x,y,0) = t(x,y)H_m\left(\sqrt{2}\frac{x}{w_{0x}}\right)H_n\left(\sqrt{2}\frac{y}{ew_{0x}}\right) \times \exp\left[-\left(\frac{x^2}{w_{0x}^2}+\frac{y^2}{e^2w_{0x}^2}\right)\right], \tag{2}$$

where the window function of the hard-edge aperture is written as

$$t(x,y) = \begin{cases} 1 & x^2+y^2 \leqslant a^2, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

According to the angular spectrum representation [1], we can express the field at the z-plane as

$$E(x,y,z) = \int\int_{-\infty}^{\infty} A(p,q)\exp[ik(px+qy+\gamma z)]\,dp\,dq, \tag{4}$$

where k is the wave number in vacuum, $A(p, q)$ is written as

$$\begin{aligned} A(p,q) &= \left(\frac{k}{2\pi}\right)^2 \int\int_{-\infty}^{\infty} E(x,y,0) \times \exp[ik(px+qy)]\,dx\,dy \\ &= \left(\frac{k}{2\pi}\right)^2 \int\int_{-\infty}^{\infty} t(x,y)H_m\left(\frac{\sqrt{2}x}{w_{0x}}\right)H_n\left(\frac{\sqrt{2}y}{ew_{0x}}\right) \\ &\quad \times \exp\left[-\left(\frac{x^2}{w_{0x}^2}+\frac{y^2}{e^2w_{0x}^2}\right)\right] \\ &\quad \times \exp[ik(px+qy)]\,dx\,dy, \end{aligned} \tag{5}$$

$$\gamma = \begin{cases} \sqrt{1-p^2-q^2} & p^2+q^2 \leqslant 1, \\ i\sqrt{p^2+q^2-1} & \text{otherwise.} \end{cases} \tag{6}$$

Since there is no circular symmetry for the elliptical Gaussian beam but there is one for the window function, we are unable to obtain the analytical result by integrating Eqs. (4) and (5) directly. In order to give the analytical expression of the propagation field, we first expand the hard-edge aperture function as the sum of complex Gaussian functions with finite terms

$$t(x,y) = \sum_{j=1}^{M} B_j \exp\left[-\frac{C_j}{a^2}(x^2+y^2)\right], \tag{7}$$

where the complex constants B_j and C_j are the expansion and Gaussian coefficients, respectively, which can be obtained by optimization computation directly [8].

Inserting Eq. (7) into (5) and using the following integral formula [9]:

$$\int_{-\infty}^{\infty} \exp\left[-\frac{(t-b)^2}{2c}\right]H_m(t)\,dt = \sqrt{2\pi c}(1-2c)^{m/2}H_m\left(\frac{b}{\sqrt{1-2c}}\right), \tag{8}$$

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