Available online at www.sciencedirect.com

SCIENCE DIRECT®

ELSEVIER

Physica A 366 (2006) 107–114

PHYSICA A

www.elsevier.com/locate/physa

Hydrodynamic model for the system of self propelling particles with conservative kinematic constraints; two dimensional stationary solutions

V.I. Ratushnaya[a], V.L. Kulinskii[b,*], A.V. Zvelindovsky[c], D. Bedeaux[a]

[a]*Colloid and Interface Science group, LIC, Leiden University, P.O. Box 9502, 2300 RA Leiden, The Netherlands*
[b]*Department for Theoretical Physics, Odessa National University, Dvoryanskaya 2, 65026 Odessa, Ukraine*
[c]*Department of Physics, Astronomy & Mathematics, University of Central Lancashire, Preston PR1 2HE, UK*

Received 7 July 2005; received in revised form 3 November 2005
Available online 1 December 2005

Abstract

In a first paper we proposed a continuum model for the dynamics of systems of self propelling particles with kinematic constraints on the velocities. The model aims to be analogous to a discrete algorithm used in works by T. Vicsek et al. [Phys. Rev. Lett. 75 (1995) 1226]. In this paper we prove that the only types of the stationary planar solutions in the model are either of translational or axial symmetry of the flow. Within the proposed model we differentiate between finite and infinite flocking behavior by the finiteness of the kinetic energy functional.
© 2005 Elsevier B.V. All rights reserved.

Keywords: Self-propelling particles; Finite-flocking behavior; Vortex

1. Introduction

The dynamics of systems of particles subjected to nonpotential interactions remains poorly understood. The absence of a Hamiltonian for such systems, which generally are far from equilibrium, hampers applying the machinery of statistical mechanics based on the Liouville equation. Many attempts have been made to investigate these systems using discrete algorithms to model this behavior. In nature there are many examples of such systems [1]. Since the discrete algorithms are hard to describe analytically it is natural also to consider continuum models of a hydrodynamic type. In standard hydrodynamics the relation between microscopic kinetics (Boltzmann-type equations) and Navier–Stokes equations is a standard topic of research [2]. For the systems of interest the construction of corresponding kinetic equations based on the specific dynamic rules and their connection with the hydrodynamics equations seems to be unknown so far and is worth studying.

There are some hydrodynamic models which are based on the extension of Navier–Stokes equation [3,4]. There the additional terms which break the Gallilean invariance of the Navier–Stokes equation were

*Corresponding author.
E-mail address: kulinskij@onu.edu.ua (V.L. Kulinskii).

0378-4371/$ - see front matter © 2005 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2005.11.002

introduced, but the specific connection with the microscopic (discrete) interactions between the particles is not established. Our continuum model is based on the equations which reflect the nonpotential character of the interactions and the presence of kinematic constraints imposed on the motion of self propelling particles.

In our first paper [5] we proposed a hydrodynamic model which can be considered to be the continuum analogue of the discrete dynamic automaton proposed by T. Vicsek et al. [6] for a system of self propelling particles. It uses the continuity equation

$$\frac{\partial n(\mathbf{r},t)}{\partial t} + \mathrm{div}(n(\mathbf{r},t)\,\mathbf{v}(\mathbf{r},t)) = 0, \tag{1}$$

which implies that the total number of particles

$$N = \int n(\mathbf{r},t)\,\mathrm{d}\mathbf{r} \tag{2}$$

is constant. The kinetic energy of a co-moving volume

$$T = \frac{1}{2}\int n(\mathbf{r},t)\mathbf{v}^2(\mathbf{r},t)\,\mathrm{d}\mathbf{r} \tag{3}$$

is also conserved

$$\frac{\mathrm{d}}{\mathrm{d}t}T = 0. \tag{4}$$

Using Eqs. (1) and (4) it can be shown that a field $\boldsymbol{\omega}$ exists such that the Eulerian velocity, $\mathbf{v}(\mathbf{r},t)$, satisfies:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{v}(\mathbf{r},t) = \boldsymbol{\omega}(\mathbf{r},t) \times \mathbf{v}(\mathbf{r},t). \tag{5}$$

This equation can be considered as the continuous analogue of the conservative dynamic rule used by T. Vicsek et al. [6].

We proposed the following "minimal" model for the field of the angular velocity $\boldsymbol{\omega}(\mathbf{r},t)$ which is linear in spatial gradients of the fields $n(\mathbf{r},t)$ or $\mathbf{v}(\mathbf{r},t)$:

$$\boldsymbol{\omega}(\mathbf{r},t) = \int K_1(\mathbf{r}-\mathbf{r}')n(\mathbf{r}',t)\,\mathrm{rot}\,\mathbf{v}(\mathbf{r}',t)\,\mathrm{d}\mathbf{r}' + \int K_2(\mathbf{r}-\mathbf{r}')\nabla n(\mathbf{r}',t) \times \mathbf{v}(\mathbf{r}',t)\,\mathrm{d}\mathbf{r}'. \tag{6}$$

The $\boldsymbol{\omega}$ has the proper pseudovector character. The averaging kernels $K_1(\mathbf{r}-\mathbf{r}')$ and $K_2(\mathbf{r}-\mathbf{r}')$ should naturally decrease with the distance in realistic models. They sample the density and the velocity around $\mathbf{r}$ in order to determine $\boldsymbol{\omega}(\mathbf{r},t)$. In the first paper we concentrated on K_1. The detailed derivation of the above equations from the discrete models based on the automaton proposed by Vicsek et al. [6,7] will be the subject of a future paper.

Note that the models based on Eqs. (1)–(6) allow solutions of uniform motion in the form of a solitary packet:

$$n(\mathbf{r},t) = n_0(\mathbf{r}-\mathbf{v}_0 t) \tag{7}$$

with $\mathbf{v}_0$ independent of position and time. The contribution to $\boldsymbol{\omega}$ due to K_1 is zero for an arbitrary density distribution n_0. The contribution due to K_2 is zero for density distributions n_0 which only depend on the position in the $\mathbf{v}_0$ direction. In this second case it follows from the continuity equation that n_0 should be everywhere constant. The density distribution n_0 should be chosen such that the number of particles, and, correspondingly, the total kinetic energy are finite. The solutions of such type also were found analytically in [8] and observed in simulations [9]. Note that such solutions exist not only in nonlocal case like in Ref. [8] but also for the local model which we consider below.

Within the first order of perturbation theory on small deviation of density and velocity fields the solitary solution given by Eq. (7) shows neutral stability; i.e., the perturbations grow linearly for small t.

We restrict our discussion to the simple case of averaging kernels, which are δ-functions:

$$K_j(\mathbf{r}-\mathbf{r}') = s_j\delta(\mathbf{r}-\mathbf{r}'), \quad \text{where } j = 1 \text{ or } 2. \tag{8}$$

Download English Version:

https://daneshyari.com/en/article/978605

Download Persian Version:

https://daneshyari.com/article/978605

Daneshyari.com