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Brownian motion of a parametric oscillator driven by asymmetric square waves, classical fluctuation squeezing and unidirectional transport of matter in dense media

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Abstract

Brownian motion of a parametric oscillator with asymmetric square waves which take a spring constant during a given length of time and suddenly change to a different value with a different length of time-duration has been investigated analytically with a view to the Paul trap in contrast to usual symmetric fields. We assume that the square wave is periodically applied in time. The first and second moments for velocity and displacement have been analytically expressed in terms of simple matrices. The stable–unstable regions are shown explicitly. We consider how the unidirectional motion which is useful for the transport of particles may be recovered by the parametric oscillation. Moreover, it is shown that the fluctuation expressed by the mean square displacement becomes small in spite of increase of the amplitude of the square waves, which is called the classical fluctuation squeezing. This fluctuation squeezing is pronounced by the asymmetry of the waves and remarkably it takes double minima at certain regions. This model provides one of simple examples that randomness creates order.
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Keywords: Brownian motion; Langevin equation; Parametric oscillator

1. Introduction

Although investigation of parametric oscillations typically represented by physical phenomena described by the Mathieu equation [1,2,5] has a long history going back to 19th century [1,3,4], it still has important problems to be worked out [5–14]. The Paul trap [15] is closely associated with the parametric oscillation, which is also useful for designing quantum computers [16]. We also note connection of the oscillation with movement of molecules using the fluctuation of external noise for useful worked by ratchet potential [17]. To shed bright light on these fields, we shall treat Brownian motion of the oscillator driven by asymmetric square waves in this paper, which has not been carried out before as far as we are aware of. The oscillator is governed

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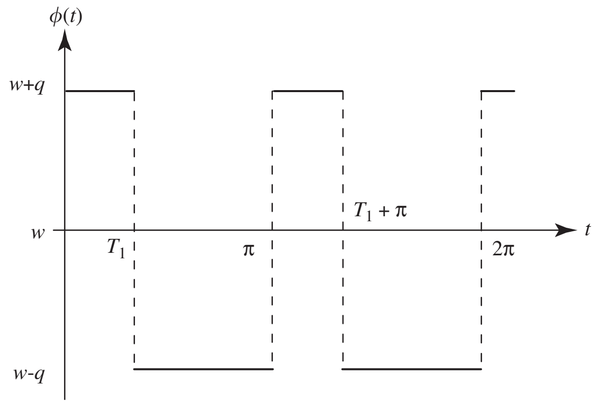

Fig. 1. Definition of square waves $\phi(t)$ with a period π.

by the following Langevin equation:

$$\ddot{x}(t) + \beta\dot{x}(t) + \phi(t)x(t) = f(t), \tag{1}$$

where $x(t)$ is the position of a particle at time t, β is a damping coefficient, $\phi(t)$ is square waves with a period π as follows and shown in Fig. 1,

$$\phi(t) = \begin{cases} w+q, & n\pi \leqslant t < n\pi + T_1 \\ w-q, & n\pi + T_1 \leqslant t < (n+1)\pi \end{cases} \quad n \in \boldsymbol{Z} \tag{2}$$

and $f(t)$ is centered Gaussian white noise with

$$\langle f(t)\rangle = 0, \quad \langle f(t_1)f(t_2)\rangle = 2\varepsilon\delta(t_1 - t_2), \tag{3}$$

where $\varepsilon = \beta k_B T$ according to the fluctuation dissipation theorem [18].

Detailed examinations on the present system will be made analytically. After calculating the first moment in the unstable region, we will determine explicitly conditions where a unidirectionally increasing displacement of particles becomes available, which is a significantly different kind of motion from that caused by ratchet potential. Namely, we show that the unidirectional transport of particles in dense media such as in liquid is possible by the deterministic motion of the parametric oscillation in the unstable region. Then with the second moment in the stable region, the mean square displacement (MSD) can become smaller than that at $q = 0$ in spite of increase of q with a fixed w, which means that although the change in amplitude of external force $-\phi(t)x(t)$ is large, fluctuation of MSD decreases. This is an interesting phenomenon observed earlier [7–10] which we call the classical fluctuation squeezing. Also, quantum mechanical behaviour was generally investigated [11]. In this study, it will be revealed that an asymmetry enhances the fluctuation squeezing. Furthermore, we will find that there are double minima for the fluctuation squeezing. In other words, one of major astonishing merits of using asymmetric $\phi(t)$ in contrast to usual symmetric one is the increase in the fluctuation squeezing by asymmetric rhythms which are entropy creating, i.e., one of simple examples that increase in entropy creates *order*.

2. Theoretical

Since Eq. (1) is a linear equation and $f(t)$ is Gaussian, the probability density $W(x, v, t)$ is also Gaussian. Hence we can write down $W(x, v, t)$ immediately as follows:

$$\begin{aligned} W(x,v,t) = \frac{1}{2\pi\sqrt{|\boldsymbol{\sigma}(t)|}}\exp\Bigg[&-\frac{\sigma_{22}(t)}{2|\boldsymbol{\sigma}(t)|}(x - \langle x(t)\rangle)^2 \\ &-\frac{\sigma_{11}(t)}{2|\boldsymbol{\sigma}(t)|}(v - \langle v(t)\rangle)^2 + \frac{\sigma_{12}(t)}{|\boldsymbol{\sigma}(t)|}(x - \langle x(t)\rangle)(v - \langle v(t)\rangle)\Bigg], \end{aligned} \tag{4}$$

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