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Diffusion of optical pulses in dispersion-shifted randomly birefringent optical fibers

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Abstract

An effect of polarization-mode dispersion, nonlinearity and random variation of dispersion along an optical fiber on a pulse propagation in a randomly birefringent dispersion-shifted optical fiber with zero average dispersion is studied. An averaged pulse width is shown analytically to diffuse with propagation distance for arbitrary strong pulse amplitude. It is found that optical fiber nonlinearity can not change qualitatively a diffusion of pulse width but can only modify a diffusion law which means that a root mean square pulse width grows at least as a linear function of the propagation distance.

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Polarization-mode dispersion (PMD), which is a pulse broadening caused by random variation of optical fiber birefringence, has recently become a major drawback in the development of new high-bit-rate optical communication systems [1–7]. Another effect, which limits bit-rate capacity, is pulse broadening caused by group-velocity dispersion (GVD). Use of a dispersion-shifted fiber with zero average GVD can reduce this effect, however, in such fibers GVD inevitably fluctuates around zero along the propagation direction [8,9] and hence pulse broadening still occurs [10,11]. Nonlinearity in optical fibers results in the coupling of both PMD and GVD effects, so in general they can not be studied separately in contrast to linear case. Linear PMD was first studied in [1,4,5] while nonlinear PMD was addressed in numerical experiments [2] and analytical studies based on a perturbation expansions around soliton solutions of deterministic equations [3,12–15]. An effect of random variation of GVD was studied in

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[10,11,16–18]. Here, an exact analytical (nonperturbartive) theory is developed for the case of fiber with random birefringence and random GVD with zero mean and arbitrarily strong nonlinearity (arbitrary pulse amplitude). No assumption like closeness to any type of soliton solution is necessary for the results of this Article to be valid. The main result is that a statistical average (over random variation of fiber parameters) of root mean square pulse width T_{RMS} grows with distance at least as a linear function of propagation distance. This means that random diffusion of optical pulse width can not be prevented by an arbitrarily strong nonlinearity. It is shown that random diffusion fundamentally limits the bit-rate capacity of an optical fiber.

Neglecting second-order GVD (dispersion slope) effects, stimulated Raman scattering and Brillouin scattering, the propagation of optical pulses in birefringent optical fibers is described by the two-component vector nonlinear Schrödinger equation (VNLS) [19,3,20,14]

$$\mathrm{i}\partial_z\eta_\alpha + \sum_{\beta=1}^{2} \Delta_{\alpha\beta}(z)\eta_\beta + \mathrm{i}\sum_{\beta=1}^{2}\tilde{m}_{\alpha\beta}(z)\partial_t\eta_\beta + d(z)\partial_t^2\eta_\alpha + \sigma(z)\tilde{N}_\alpha(\eta) = \mathrm{i}G(z)\eta_\alpha, \qquad (1)$$

where z is the propagation distance along an optical fiber, η_1 and η_2 correspond to the complex amplitudes of two orthogonal linear polarizations, $t \equiv \tau - z/c_1$ is the retarded time and τ is the physical time, c_1 is the speed of light, and $d(z)$ is the dispersion, which is related to first-order GVD β_2 as $d(z) = -\frac{1}{2}\beta_2(z)$. The right hand side (r.h.s.) of Eq. (1) describes linear losses and amplifiers

$$G(z) \equiv \left(-\gamma + [\exp(z_a\gamma) - 1]\Sigma_{k=1}^{N}\delta(z - z_k)\right),$$

$\sigma = (2\pi n_2)/(\lambda_0 A_{\mathrm{eff}})$ is the nonlinear coefficient, n_2 is the nonlinear refractive index, $\lambda_0 = 1.55$ μm is the carrier wavelength, A_{eff} is the effective fiber area, $z_k = kz_{\mathrm{a}}$ ($k = 1,\dots,N$) are the amplifier locations, z_{a} is the amplifier spacing, and γ is the loss coefficient. Distributed amplification can be also included by adding z-dependence into γ. Properties of fiber can be different along optical line, e.g., A_{eff} could be different if line consists of several pieces of fiber with different cross section, and, respectively, coefficient σ generally depends on z. In a similar way, all parameters of fiber, like $d(z)$ also depend on z.

The self-conjugated matrices $\hat{\Delta}(z)$ and $\hat{\tilde{m}}(z)$ describe, respectively, the differences in wave vectors and the anisotropy of the group velocities of the two modes corresponding to the two different polarizations. Both matrces $\hat{\Delta}$ and $\hat{\tilde{m}}$ are made traceless. The trace of the matrix $\hat{\Delta}$ is excluded by a phase transformation $\tilde{\eta} \to \eta\exp(\mathrm{i}\phi_0 z)$. The trace of the matrix $\hat{\tilde{m}}$ is zero because Eq. (3) is written in a frame moving with average group velocity (note that group velocity is generally z-dependent). It is assumed in Eq. (3) that the dispersion $d(z)$ and nonlinearity are isotropic because their anisotropy is usually negligible in optical fibers. Vector $\tilde{\mathbf{N}} = (\tilde{N}_1, \tilde{N}_2)^{\mathrm{T}}$, which represents the contribution of Kerr nonlinearity, is given by

$$\begin{aligned}\tilde{N}_1(\Psi) &= \left[\left(|\Psi_1|^2 + \frac{2}{3}|\Psi_2|^2\right)\Psi_1 + \frac{1}{3}\Psi_2^2\Psi_1^*\right],\\ \tilde{N}_2(\Psi) &= \left[\left(\frac{2}{3}|\Psi_1|^2 + |\Psi_2|^2\right)\Psi_2 + \frac{1}{3}\Psi_1^2\Psi_2^*\right]\end{aligned} \qquad (2)$$

see [19,3].

The change of variables $\xi = \eta\mathrm{e}^{-\int_0^z G(z')\,\mathrm{d}z'}$ (see e.g. [21,22]) removes r.h.s. of Eq. (1) and gives

$$\mathrm{i}\partial_z\xi_\alpha + \sum_{\beta=1}^{2} \Delta_{\alpha\beta}(z)\xi_\beta + \mathrm{i}\sum_{\beta=1}^{2}\tilde{m}_{\alpha\beta}(z)\partial_t\xi_\beta + d(z)\partial_t^2\xi_\alpha + c(z)\tilde{N}_\alpha(\xi) = 0, \qquad (3)$$

where $c(z) \equiv \sigma(z)\exp(2\int_0^z G(z')\,\mathrm{d}z')$. Thus, all linear fiber losses and amplifications are included into coefficient $c(z)$.

The isotropic case, which corresponds to zero matrices $\hat{\Delta} = \hat{\tilde{m}} = \hat{0}$, allows a solution of Eq. (3) with constant polarization, e.g., $\xi_1 \neq 0$, $\xi_2 = 0$. Components of matrices $\hat{\Delta}$ and $\hat{\tilde{m}}$ fluctuate strongly as functions of distance z. Fluctuations correspond to violation of circular symmetry of the fiber. The matrices $\hat{\Delta}$ and $\hat{\tilde{m}}$ change in optical fibers with time on a scale of few hours because of environmental fluctuations, however, for typical optical pulse duration (10 ps), one can consider $\hat{\Delta}$ and $\hat{\tilde{m}}$ as functions of z only. It means that disorder is frozen in the fiber. The matrix $\hat{\Delta}$ gives the leading order contribution in Eq. (3) because a typical beat length z_{beat} (typical length at which a relative phase shift between two polarizations