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Thermal entanglement in Spin-1 biparticle system

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Abstract

The thermal entanglement in a two-qubit Spin-1 system with two spins coupled by exchange interaction is investigated in terms of the measure of entanglement called "negativity". It is found that the thermal entanglement exists and is symmetric for both ferromagnetic and antiferromagnetic exchange couplings. Moreover, the critical temperature at which the negativity vanishes increases with the exchange coupling constant J. From the temperature and magnetic field dependences we demonstrate that the temperature and the magnetic field can affect the feature of the thermal entanglement significantly.

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Entanglement as a key concept in quantum information processing (QIP) [1–3] has attracted a lot of attention both experimentally and theoretically in recent years [4]. Since the entanglement is fragile, the problem of how to create stable entanglement remains a main focus of recent studies in the field of quantum information processing. The thermal entanglement, which differs from the other kinds of entanglements by its advantages of stability for the reduction in entanglement of an entangled state due to various sources of decoherence and in entanglement in time due to thermal interactions are absent as the entanglement at finite temperature takes thermal decoherence into account implicitly, requires neither measurement nor controlled switching of interactions in the preparing process, and hence becomes an important quantity of systems for the purpose of quantum computing.

The system of atoms in optical lattices is among the promising candidates for quantum information processing. It may take the advantage of the

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technology used in atom optics and laser cooling based on the optical manipulation of atoms [5]. Besides, it also holds the merit of eventual possibility to scale, parallelize and miniaturize the device in QIP.

The thermal entanglement has been extensively studied for various systems including isotropic [6–9] and anisotropic [10] Heisenberg chains, Ising model in an arbitrarily directed magnetic field [11], and cavity-QED [12] since the seminal works by Arnesen et al. [6] and Nielsen [13]. Based on the method developed in the context of quantum information, the relaxation of a quantum system towards the thermal equilibrium is investigated [14] and provides us an alternative mechanism to model the spin systems of the Spin-$\frac{1}{2}$ case for the approaching of the thermal entangled states [6–10]. The development of laser cooling and trapping provides us more ways to control the atoms in traps. Indeed, we can manipulate the atom-atom coupling constants and the atom number in each lattice well with a very well accuracy [15,16]. Our system consists of two wells in the optical lattice with one Spin-1 atom in each well. The lattice may be formed by three orthogonal laser beam, and we may use an effective Hamiltonian of the Bose–Hubbard form [17] to describe the system. The atoms in the Mott regime make sure that each well contains only one atom. For finite but small hopping term t, we can expand the Hamiltonian into powers of t and get [16]

$$H = \epsilon + J(S_1 \cdot S_2) + K(S_1 \cdot S_2)^2, \tag{1}$$

where $J = -2t^2/U_2$, $K = -2t^2/3U_2 - 4t^2/U_0$ with t the hopping matrix elements, and $\epsilon = J - K \cdot U_s (s = 0,2)$ represents the Hubbard repulsion potential with total spin s, a potential U_s with $s = 1$ is not allowed due to the identity of the bosons with one orbital state per well, since term ϵ contains no interaction, we can ignore it in the following discussions and it would not change the thermal entanglement. For simplification, $J \gg K$ is assumed and the nonlinear couplings is ignored. So the Hamiltonian Eq. (1) becomes

$$H = J(S_1 \cdot S_2). \tag{2}$$

We begin with the two-qubit model in the absence of the external magnetic field

$$H = J(S_1^x S_2^x + S_1^y S_2^y), \tag{3}$$

in which the neglected exchange coupling term along the z-axis is assumed to be much smaller than the coupling in the x–y plane. Where S^α ($\alpha = x,y$) are the spin operator, J is the strength of Heisenberg interaction. With the help of raising and lowering operators $S_n^\pm = S_n^x \pm iS_n^y$, the Hamiltonian H is rewritten as

$$H = \frac{J}{2}(S_1^+ S_2^- + S_1^- S_2^+). \tag{4}$$

To evaluate the thermal entanglement we first of all find the eigenvalues and the corresponding eigenstates of the Hamiltonian Eq. (4) which are seen to be

$$H|1,1\rangle = 0, H|-1,-1\rangle = 0,$$

$$H|\Psi_1\rangle = 0, H|\Psi_2^\pm\rangle = \pm\frac{J}{2}|\Psi_2^\pm\rangle,$$

$$H|\Psi_3^\pm\rangle = \pm\frac{J}{2}|\Psi_3^\pm\rangle,\ H|\Psi_4^\pm\rangle = \pm\frac{J}{\sqrt{2}}|\Psi_4^\pm\rangle, \tag{5}$$

where

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|1,-1\rangle - |-1,1\rangle),$$

$$|\Psi_2^\pm\rangle = \frac{1}{\sqrt{2}}(|0,-1\rangle \pm |-1,0\rangle),$$

$$|\Psi_3^\pm\rangle = \frac{1}{\sqrt{2}}(|1,0\rangle \pm |0,1\rangle),$$

$$|\Psi_4^\pm\rangle = \frac{1}{\sqrt{2}}|-1,1\rangle \pm |0,0\rangle + \frac{1}{\sqrt{2}}|1,-1\rangle). \tag{6}$$

The density operator at thermal equilibrium $\rho(T) = \exp(-\beta H)/Z$, where $Z = Tr[\exp(-\beta H)]$ is the partition function and $\beta = 1/k_B T$ (k_B is Boltzmann's constant being set to be unit $k_B = 1$ hereafter for the sake of simplicity), can be expressed in terms of the eigenstates and the corresponding eigenvalues as

$$\begin{aligned}\rho(T) = \frac{1}{Z}\{&|\Psi_1\rangle\langle\Psi_1| + |-1,-1\rangle\langle -1,-1| + |1,1\rangle\langle 1,1| \\ &+ \exp[m]|\Psi_2^+\rangle\langle\Psi_2^+| + \exp[-m]|\Psi_2^-\rangle\langle\Psi_2^-| \\ &+ \exp[m]|\Psi_3^+\rangle\langle\Psi_3^+| + \exp[-m]|\Psi_3^-\rangle\langle\Psi_3^-| \\ &+ \exp[\sqrt{2}m]|\Psi_4^+\rangle\langle\Psi_4^+| + \exp[-\sqrt{2}m] \\ &\times |\Psi_4^-\rangle\langle\Psi_4^-|\},\end{aligned} \tag{7}$$

with the partition function seeing to be $Z = 3 + 4\cos h[m] + 4\cos h[\sqrt{2}m]$ and $m = J/2T$.